A NOVEL ANALYTIC HIERARCHY PROCESS TECHNIQUE FOR LARGE AND FUZZY CRITERIA DECISION MAKING PROBLEMS

By
MR. Peerapop JOMTONG

A Thesis Submitted in Partial Fulfillment of the Requirements
for Doctor of Philosophy ENGINEERING MANAGEMENT
Department of INDUSTRIAL ENGINEERING AND MANAGEMENT
Silpakorn University
Academic Year 2022

เทคนิคกระบวนการลำดับชั้นเชิงวิเคราะห์สำหรับปัญหาการตัดสินใจเกณฑ์ขนาดใหญ่และคลุมเครือ

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Title	A Novel Analytic Hierarchy Process Technique for Large and Fuzzy Criteria Decision Making Problems
By	MR. Peerapop JOMTONG
Field of Study	ENGINEERING MANAGEMENT
Advisor	Associate Professor Dr. Choosak Pornsing

Faculty of Engineering and Industrial Technology, Silpakorn University in Partial Fulfillment of the Requirements for the Doctor of Philosophy

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This dissertation is a study of the Analytic Hierarchy Process (AHP) and is divided into two main parts. In the first part, the researcher requires the development of a new comparison procedure of an analytic hieratical process to make it convenient to use the AHP analysis to apply on cases with large criteria. The proposed AHP and the scoring methods will be improved to make it simple for experts. The method is called “Normalize function-based scaling AHP” The researcher proposed a novel technique by borrowing the idea of the Likert scale but employing a 1 to 9 scale. By comparing the proposed method with the classic AHP with a clustering technique, the proposed method yielded the same conclusion as the classic AHP while requiring significantly less effort.

Furthermore, the threshold of decision changing was not a substantial discrepancy. In the second part, this research wants to increase the performance of FAHP methods. It is to compare 2 decision-making methodologies, classic AHP and FAHP (Triangle, Trapezoidal) in the case of choosing the preferable medical devices using the weighing results and consistency ratio values on the same data in the case of medical device suppliers. The result, in case, one needs the calculation with less bias, a user should consider FAHP (Triangle) method, as FAHP (Triangle) allows the user to detect and analyze consistency ratio more rapidly but one must accept that it involves more complicated calculation which is considerably recommended for the amateur assessor with an authority to approve such vendor, while classic AHP is suitable for assessors with excessive experience.

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CHAPTER 1

Introduction

1.1 Motivation

Decision-making is necessary for daily life. It is effective, it must a logical decision to contemplate the good and bad results of alternatives, and information is a benefit for the organization or the community as a whole, and consistent with the rules and decisions that are well-timed [1, 2].

The multi-criteria decision-making (MCDM) is a decision of factors facing different and inconsistent units of measuring [3]. Hwang and Masud [4] summarize that all MCDM problems share the following common characteristics:

> "Multiple-criteria: each problem has multiple-criteria, conflict among criteria: Multiple-criteria often conflict with each other. Incommensurable units: Multiple-criteria may have different units of measurement. Design/selection: Solutions to an MCDM problem are either to design the best alternative(s) or to select the best one(s) among a pre-specified finite set of alternatives."

The decision problem with benefits affects the decision-making regardless of the basis of cause and effect which directly or indirectly affects the decision-making [5, 6]. Also, the decisions are made to fail due to unstable, incomplete information and the decision-making under risk. The decision-maker will have to guess the opportunity or possibility based on experience, and the decision-making to consider the highest return and opportunity for the selection [7]. We need to analyze future trends, and the decision in advance to avoid future problems. In particular, the complicated decision have related rules, e.g., we make decisions on which building to buy, which warehouse to choose, how to design an optimal investment strategy to balance profit and risk, etc.

The multi-criteria decision-making can be used exclusively techniques, e.g., Jasiński et al. [8] use of MCDM methods to assign a risk class to each material for ELECTRE-TRI based. Fazeli et al. [9] use the MCDM framework to link the energy system model for electric vehicle (EV) adoption in Iceland, and the most effective policy measure in increasing the adoption of EVs. Sakthivel et al. [10] present the MCDM technique and the analytical network process for the selection of optimal fuel blend in fish oil biodiesel for the internal combustion engine. Bal et al. [11] present the application of data envelopment analysis of MCDM in which to reduce the maximal quantity among all variable deviations and to reduce the summation deviations. Fan et al. [12] use MCDM problems and a dominance-based rough set approach to introduce a set of decision rules from sample decisions which decision-makers can advise on the new decision-making environment. De Farias Aires et al. [13] make use of the ELECTRE-TRI multi-criteria decision-making method in retail enterprise's distribution centers to assist in investment decisions which are implemented in a new technological structure for use in the company's centralized data processing system. Jayaraman et al. [14] propose an MCDM using a goal programming model for strategic planning and resource allocation to expand and implement responsible strategies for sustainability. The problems in construction management were analyzed, solved, and discussed by the combination of MCDM and analytic hierarchy process (AHP) approaches.

The AHP is a structured technique for collecting and analyzing complex decisions, based on mathematics and psychology. It is developed by Thomas L. Saaty in the 1970s. It is a method of "measurement through pairwise comparisons and depends on the decisions of experts to derive priority scales" [15]. AHP has been one of using multiple-criteria decision-making tools and has been extensively studied. It has an extensive variety of applications like resource allocation of business or public policy, strategic planning, source selection, program selection, and task priority [16]. Presently, AHP has been used in conjunction with fuzzy, called fuzzy analytic hierarchy process (FAHP). Jayawickrama et al. [17] present a generic model to evaluate the sustainability performance of a manufacturing plant using FAHP. This tool helps to resolve a variation point or a variability that is used to evaluate the feasibility study of the plant operation. Kaganski et al. [18] make use of the FAHP as a tool for the prioritization of key performance indicators based on SMARTER criteria and 13 KPIs, the weights for the SMARTER criteria are developed. Radziszewska [19] proposes supporting partnering relation management in the implementation of construction projects using FAHP as such an adjustment is likely to be highly advantageous to the implementation of a construction project in terms of its duration, cost, quality, and safety.

Based on the primary literature review of this study, it is found that the pairwise comparison also has a point that can be developed better. In the case of multiple-criteria, it may cause the experts confused by the double scoring. Garbuzova-Schlifter et al. [20] present an AHP-based risk analysis of energy efficiency projects in Russia with 8 main criteria. There are tool criteria 28 pairwise comparisons and 29 sub-criteria pairwise comparisons; accordingly, resulting in errors easily because of confusion.

From the primary study of related literature, it is found that a number of studies deploy fuzzy functions combine with AHP which is called FAHP. Moreover, as there are many types of fuzzy function, the most popular type is the triangular function because it is easy to make understand and improve the accuracy of pairwise comparisons [21-23]. Followed by the trapezoidal function [24] which is applied to check the consistency with the centric consistency index using the extent analysis method of trapezoidal. And the gaussian function [25] is developed to gaussian FAHP to execute gaussian fuzzy numbers to eliminate the case of zero weights. The research outcomes stemming from the gaussian FAHP are produces more accurate and realistic results than the conventional FAHP methods. Expert consistency prioritization is conducted for expertise differences instead of assuming identical experts. The trapezoidal function and the gaussian function have been used recently, but they are still less popular nowadays. Therefore, in this study, the researcher is interested in studying what type of FAHP which is suitable for specific tasks that will make the most effective decision-making.

In the first part of this study, requires to development of a new comparison procedure of an analytic hieratical process in order to be convenient for using AHP analysis apply to cases with large criteria. The proposed AHP will be improved and the scoring methods to make it easier for experts. In the second part, there are many fuzzy functions; for example, triangular function, trapezoidal function, r-functions, l-functions, gaussian function, generalized bell functions, sigmoid functions, etc, the problem at hand is which function is suitable for a specific AHP based on the decision problem. On the other words, the problem be accordant with FAHP and the function will be the most exact for a problem. Therefore, this research wants to increase the performance of

FAHP methods. In the last part, applying the results of the scaling score for large criteria of AHP and the usage of each type of fuzzy analytic hierarchy process used to solve problems in the engineering case study will be conducted by using the proposed techniques.

1.2 Research Objective

1. To propose a scaling score method for large criteria decision-making problem.

2. To explore the knowledge of selecting fuzzy functions on FAHP.

3. To apply the proposed techniques on an engineering decision-making problem which comes up with large decision-making criteria, and some crisp data.

1.3 Research Contributions

The main contributions of this thesis are:

1. Receive a new procedure for expert scorings.

2. Able to recommend using FAHPs in practice.

3. Receive effective decision-making tools for implementation.

CHAPTER 2

Literature Review

2.1 Analytic Hierarchy Process

2.1.1 Motivation

The analytic hierarchy process (AHP) is a multi-criteria decision-making approach that Thomas Saaty invented in the 1980s. It is the best way to decide among the complex criteria structure in different levels. It is the selection of priority criteria by pairwise comparisons from all priority criteria instead of the numerical scoring based on satisfaction. AHP also provides methods for measuring and interpreting the consistency of decision-makings to mathematically precise results [26].

AHP is a suitable tool for group decision-making to achieve cooperation in decision-making and acceptance by the group. To be more accurate using AHP, a decision maker must determine the problem or purpose of the decision-making. Next, she must study the criteria related to the objective set and compare the decision factors then find the best alternative [27]. From past to present, AHP has been used for various tasks in the areas of military, aviation, education, energy, industry, healthcare, business, and others for the best decision-making for the organization.

2.1.2 Analytic hierarchy process and its extensions

A book authored by Dr.Saaty, The Analytic Hierarchy Process for Decisions in a Complex World, describes the procedure of AHP which is divided into five steps, as follows [28].

Step 1: Define the goal and criteria for decision-making

Group the problem components into levels as follows:

- The top level is the decision-making goal.
- Level 2 is the criteria.
- Level 3 is the sub-criteria.
- The last level is the choice.

As shown in Fig.1.

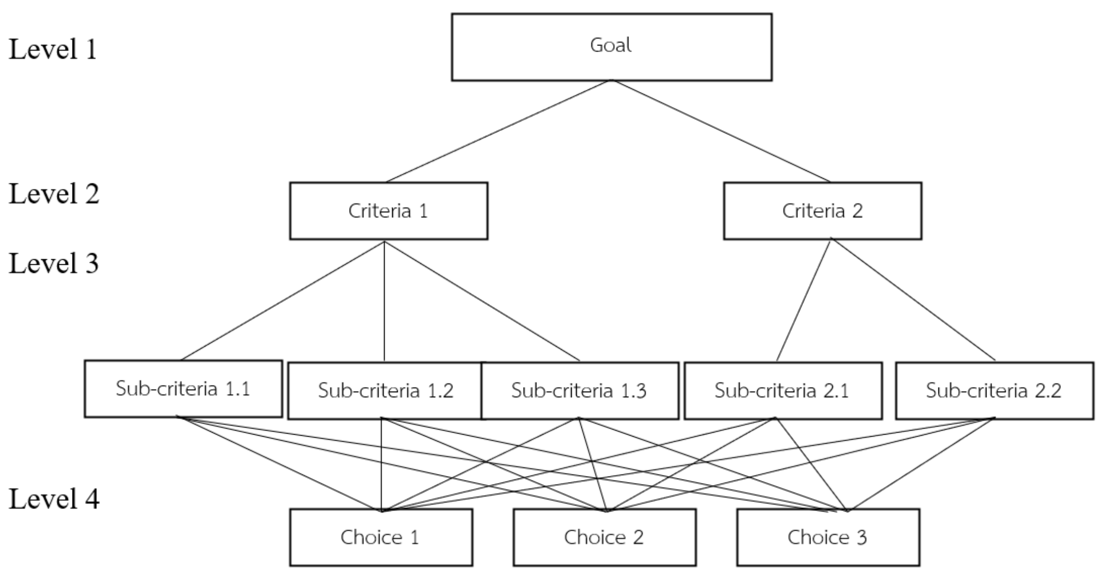

Fig. 1 Structured AHP model

Step 2: Operate pairwise comparison.

Each level compares the importance of various criteria at the same level. The comparison of alternative criteria is analyzed in pairs as shown in Table 1.

Table 1 Fundamental scale of Thomas L. Saaty

Verbal Judgments	Intensity of Importance
Equal importance	1
Moderate importance	3
Strong importance	5
Very strongly importance	7
Extreme importance	9
Intermediate values between the two adjacent judgments	2, 4, 6, 8

source: Saaty [28]

Pairwise comparison matrices have been operated to compare each element of the hierarchy structure as shown in Eq. (2.1).

$$A = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \tag{2.1}$$

Step 3: Estimate the relative weights.

The eigenvalue method computes the relative weights of elements in each pairwise comparison matrix. The relative weights (W) of matrix A are obtained from Eq. (2.2).

$$(A - \lambda_{max} I)W = 0 \tag{2.2}$$

where λ_{max} = the biggest eigenvalue of matrix A

I = unit matrix.

Step 4: Check the consistency

The Consistency Ratio (CR) of matrices is estimated to ensure that the judgments of decision-makers are consistent. The consistency ratio is computed as shown in Eq. (2.3).

$$\text{CI} = \frac{(\lambda_{max} - n)}{(n-1)} \quad (2.3)$$

where CI = consistency index

n = number of elements in the matrix

Next step

$$\text{CR} = \frac{\text{CI}}{\text{RI}} \quad (2.4)$$

where CR = consistency ratio

RI = random index computed for matrices that depend of n

Table 2 List of RI values

n	3	4	5	6	7	8	9	10	11	12	13
RI	0.58	0.89	1.12	1.24	1.33	1.40	1.45	1.49	1.51	1.54	1.56

source: Alonso & Lamata [29]

If the consistency ratio is less than or equal to 0.01, the decision is acceptable. However, if it is not, the analyst must redo the whole process [30].

Step 5: Result of the overall rating.

Finally, the criteria are ordered with the weights decreasingly. the most important criterion has the largest weight. On the other hand, the least important criterion has the smallest weight.

2.1.3 Applications of analytic hierarchy process

Aşchilean et al. [31] present the method of selecting the optimal technology to rehabilitate water distribution systems in Romania using the AHP.

Step 1: Identify the problem

They select the optimum technology for the rehabilitation of pipes from the domestic water supply system.

Step 2: Determine the decision criteria

They use seven decision criteria as shown in Table 3.

Table 3 The set of decision criteria

Criterion	Type	Description
Diameter of the pipe (C_1)	Maximize	It is advisable to select an alternative that can be used for the entire range of pipes used in water distribution networks.
Length of the pipe (C_2)	Maximize	It is advisable to select an alternative that can be used for the longest possible pipelines.
Period of time required for installation (C_3)	Minimize	It is preferable for installation be as quick as possible.
Lifespan ratio between the rehabilitated pipe and the not rehabilitated pipe (C_4)	Maximize	The lifespan of the rehabilitated pipe must be higher than the lifespan of the replaced pipe.
Pressure losses (C_5)	Minimize	The pressure losses should be as low as possible.

Price (C_6)	Minimize	The price for replacing the pipes should be as low as possible.
Installation conditions (C_7)	Minimize	The alternative should not set special installation conditions.

source: Aşchilean et al, [31]

Step 3: Determine the alternatives

In this study, detailed information on 10 rehabilitation technologies, as shown in Table 4. The structure of the AHP model for rehabilitation technology selecting is shown in Fig. 2.

Table 4 The alternative

Alternative is symbol	Alternative name
A_1	Compact Pipe
A_2	Slipline
A_3	Subline
A_4	Swagelining
A_5	CIPP (Cured in place pipe)
A_6	GFK Liner
A_7	Berstlining
A_8	Pilot Pipe
A_9	Microtunneling
A_{10}	Open cut

source: Aşchilean et al, [31]

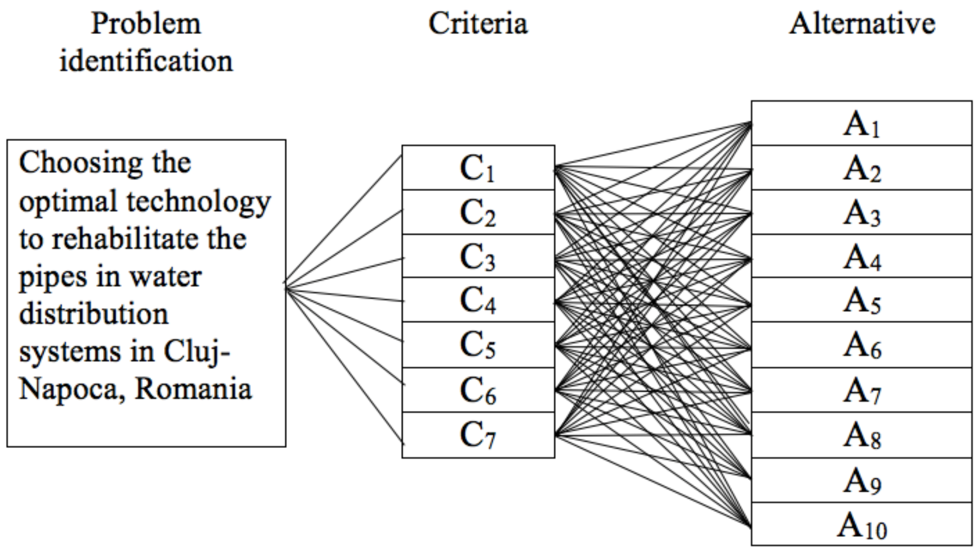

Fig. 2 Structure AHP model for rehabilitation technologies

Step 4: Determine the relative weight of the criteria

In Table 5, they present the values of the comparisons among criteria, using the fundamental scale of Thomas L. Saaty [28].

Table 5 Values of comparisons between criteria

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	1	1/3	1	1/3	1/5	1/5	1/3
C_2	3	1	3	1	1/3	1/3	1
C_3	1	1/3	1	1/3	1/5	1/5	1/3
C_4	3	1	3	1	1/3	1/3	1
C_5	5	3	5	3	1	1	3
C_6	5	3	5	3	1	1	3
C_7	3	1	3	1	1/3	1/3	1

source: Aşchilean et al, [31]

Step 5: Normalize the comparisons among criteria.

The pairwise comparison between criteria is transformed in weights based as shown in Table 6.

Table 6 Values of comparisons between criteria

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Total	Medium value
C_1	0.05	0.03	0.05	0.03	0.06	0.06	0.03	0.32	0.045
C_2	0.14	0.10	0.14	0.10	0.10	0.10	0.10	0.79	0.113
C_3	0.05	0.03	0.05	0.03	0.06	0.06	0.03	0.32	0.045
C_4	0.14	0.10	0.14	0.10	0.10	0.10	0.10	0.79	0.113
C_5	0.24	0.31	0.24	0.31	0.29	0.29	0.31	2.00	0.285
C_6	0.24	0.31	0.24	0.31	0.29	0.29	0.31	2.00	0.285
C_7	0.14	0.10	0.14	0.10	0.10	0.10	0.10	0.79	0.113
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00	7.00	1.00

source: Aşchilean et al, [31]

Step 6: Check the Consistency Ratio (CR)

Matrices are estimated to ensure that the judgments of decision-makers are consistent. They have seven decision criteria in this case study, then, if n = 7 thus RI = 1.33 and eigenvalue (λ_{max}) = 7.16. The consistency ratio is computed as shown below.

$$\text{CI} = \frac{(7.16-7)}{(7-1)} = 0.027$$

$$\text{CR} = \frac{0.027}{1.33} = 0.02$$

As the value of CR is less than 0.1, the decision criteria matrix is consistent.

Step 7: Determine the global priority

Table 7 and Fig. 3, present the determining global priority value of each alternative.

Table 7 Global priority value of the alternatives

Alternative symbol	Alternative name	Total score	Place
A_2	Slipline	0.1527	1
A_1	Compact pipe	0.1339	2
A_3	Subline	0.1134	3
A_9	Microtunneling	0.1007	4
A_8	Pilot Pipe	0.0972	5
A_7	Berstlining	0.0872	6
A_{10}	Open cut	0.0860	7
A_6	GFK Liner	0.0819	8
A_5	Cured-in-place pipe (CIPP)	0.0736	9
A_4	Swagelining	0.0733	10

source: Aşchilean et al, [31]

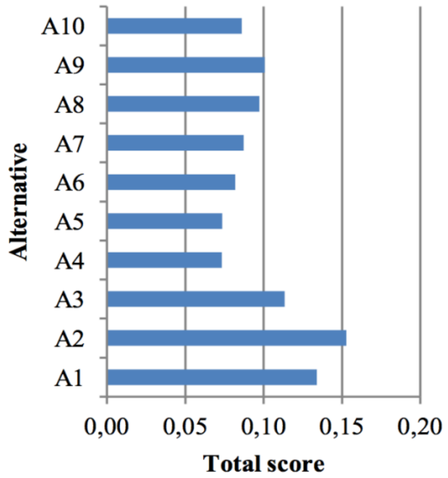

Fig. 3 Global priority value of the alternatives.
source: Aşchilean et al, [31]

The study of Aşchilean et al,[31] shows that AHP can help prioritize factors used in selecting the optimal technology to rehabilitate the pipes. The researchers found that factor A_2 has the highest weight of 15.27 %, which is clearly expressed, as well as mathematical techniques to achieve acceptance in the decision making. The study are 7 main criteria. The experts must conduct the comparison 21 times. For 10 selected alternatives of each criterion, the experts must conduct the comparison 315 times. If combine the comparison in total, it will be 336-time comparisons needed that the experts need to undergo, which is complex.

Breaz et al. [32] present the method of selecting industrial robots for the milling process. They propose a method based on the analytic hierarchy process and Quality Function Deployment (QFD).

For this approach, medium-size serial industrial robots, which can perform milling operations are taken into consideration. The characteristics of the analyzed robotic systems (R1, R2, R3) are shown in Table 8.

Table 8 The considered robotic systems

	Kinematic structure	Load capacity (kg)	Reach (mm)	Weight (kg)	Repeatability (mm)	Power consumption (KW)	Service points
R1	Serial, 6 dof	16	1611	235	±0.05	8.8	One office, branch of a reseller from abroad
R2	Serial, 6 dof	20	1811	250	±0.04	1	Only abroad (no offices in the country)
R3	Serial, 6 dof	20	1550	380	±0.03	0.67	Two offices in the country, national reseller

source: Breaz et al, [32]

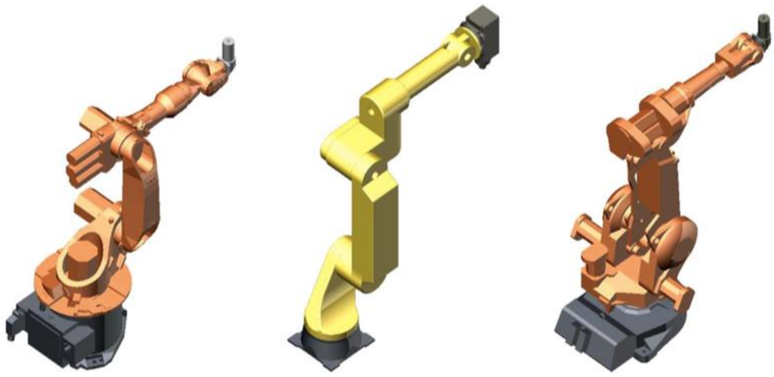

Fig. 4 The 3 geometric and kinematic models of the R1, R2, R3 robotic.
source: Breaz et al, [32]

Table 9 presents the determining of decision criteria. They use seven decision criteria as shown below.

Table 9 The criteria

Criterion	Description
C_1	load capacity (payload), defined as the maximum weight that the robot can manipulate at the level of the end-effector (it also includes the weight of the milling unit).
C_2	reach, defined as the maximum distance from the center of the robotic structure to the fullest extension of the robotic component which carries the end effector.
C_3	weight, defined as the total weight of the robotic structure.
C_4	repeatability, defined as the positioning accuracy of the end effector at a target programmed point for a given number of repetitions.
C_5	power consumption, defined as the total power required by the robotic structure.

Criterion	Description
C_6	Dexterity.
C_7	service, defined as the easiness of receiving qualified service within the country in which the robotic structure is used.

source: Breaz et al, [32]

Table 10 presents the values of the comparisons among criteria, using the fundamental scale of Saaty, T. L. [28].

Table 10 Pairwise Comparison

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	1	3	7	3	3	5	3
C_2	1/3	1	2	1/3	2	3	1/3
C_3	1/7	½	1	1/3	1/5	1/3	1/5
C_4	1/3	3	3	1	3	5	3
C_5	1/3	½	5	1/3	1	3	1/2
C_6	1/5	1/3	3	1/5	1/3	1	1/3
C_7	1/3	3	5	1/3	2	3	1

source: Breaz et al, [32]

The next step computes the eigenvector (w_i), using the formula presented below:

$$w_{ij} = \frac{\sum_{i=1}^{n} b_{ij}}{n} \tag{2.5}$$

Table 11 Compute weight

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	w
C_1	0.3737	0.2647	0.2692	0.5422	0.2601	0.2459	0.3586	0.3306
C_2	0.1246	0.0882	0.0769	0.0602	0.1734	0.1475	0.0398	0.1015
C_3	0.0534	0.0441	0.0385	0.0602	0.0173	0.0164	0.0239	0.0363
C_4	0.1246	0.2647	0.1154	0.1807	0.2601	0.2459	0.3586	0.2214
C_5	0.1246	0.0441	0.1923	0.0602	0.0867	0.1475	0.0598	0.1022
C_6	0.0747	0.0294	0.1154	0.0361	0.0289	0.0492	0.0398	0.0534
C_7	0.1246	0.2647	0.1923	0.0602	0.1734	0.1475	0.1195	0.1546

source: Breaz et al, [32]

The next step is the consistency ratio. To do so, matrices are estimated to ensure that the judgments of decision-makers are consistent. They have seven decision criteria in this case study, then, if n = 7 thus RI = 1.33 (as shown in Table 12) and eigenvalue (λ_{max}) = 7.6546. The consistency ratio is computed as shown below.

Table 12 List of RI values

n	3	4	5	6	7	8	9	10	11	12	13
RI	0.58	0.89	1.12	1.24	1.33	1.40	1.45	1.49	1.51	1.54	1.56

source: Alonso, J., & Lamata, T, [29]

$$CR = \frac{\lambda_{max} - n}{RI(n-1)} = 0.082$$

As the value of CR is less than 0.1, the decision criteria matrix is consistent.

The evaluation of the three robotic structures, with respect to the seven criteria, taken into consideration must be unfolded. The evaluation for each criterion is shown in Table 13-19.

Table 13 Comparison of the three robotic structures with regards of C1

C1	R1	R2	R3	W
R1	1	1/3	1/3	0.1428
R2	3	1	1	0.4286
R3	3	1	1	0.4286

source: Breaz et al, [32]

Table 14 Comparison of the three robotic structures with regards of C2

C2	R1	R2	R3	W
R1	1	1/3	2	0.2518
R2	3	1	3	0.5889
R3	1/2	1/3	1	0.1593

source: Breaz et al, [32]

Table 15 Comparison of the three robotic structures with regards of C3

C3	R1	R2	R3	W
R1	1	2	5	0.5559
R2	1/2	1	5	0.3537
R3	1/5	1/5	1	0.0904

source: Breaz et al, [32]

Table 16 Comparison of the three robotic structures with regards of C4

C4	R1	R2	R3	W
R1	1	1/3	1/5	0.1062
R2	3	1	1/3	0.2605
R3	5	3	1	0.6334

source: Breaz et al, [32]

Table 17 Comparison of the three robotic structures with regards of C5

C5	R1	R2	R3	W
R1	1	1/7	1/9	0.0611
R2	7	1	5	0.6582
R3	9	1/5	1	0.2807

source: Breaz et al, [32]

Table 18 Comparison of the three robotic structures with regards of C6

C6	R1	R2	R3	W
R1	1	3	3	0.5889
R2	1/3	1	1/2	0.1593
R3	1/3	2	1	0.2518

source: Breaz et al, [32]

Table 19 Comparison of the three robotic structures with regards of C7

C7	R1	R2	R3	W
R1	1	3	1/3	0.2605
R2	1/3	1	1/5	0.1062
R3	3	5	1	0.6334

source: Breaz et al, [32]

Table 13-19 are the comparison of each criterion with R1 R2 R3 robotic to determine the importance weight.

In the next step, they bring the weight of criteria importance from Table 11 to compute with Table 13-19 in order to find the most suitable value, according to the following relation:

$$X = Cw$$

$$= \begin{bmatrix} 0.1428 & 0.1062 & 0.2605 & 0.0611 & 0.2518 & 0.5889 & 0.5559 \\ 0.4286 & 0.2605 & 0.1062 & 0.6582 & 0.5889 & 0.1593 & 0.3537 \\ 0.4286 & 0.6334 & 0.6334 & 0.2807 & 0.1593 & 0.2518 & 0.0904 \end{bmatrix} \begin{bmatrix} 0.3306 \\ 0.1015 \\ 0.0363 \\ 0.2214 \\ 0.1022 \\ 0.0534 \\ 0.1546 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2241 \\ 0.4411 \\ 0.3348 \end{bmatrix}$$

From the results, it can be concluded that the weight of R2 is the highest.

As shown above, AHP is a tool for decision-making tool by creating a hierarchy chart to understand the structure of goals and criteria. There is still a pairwise comparison and analysis the consistency of cause and effect using mathematical principles, making AHP easy to understand and use. The study are 7 main criteria, The experts must conduct the comparison 21 times. For 3 selected alternatives of each criterion, the experts must conduct the comparison 21 times. If combine the comparison in total, it will be 42 times.

2.2 The Fuzzy Theory

2.2.1 Fuzzy set

The fuzzy set becomes a significant technique for artificial intelligence since the fuzzy set allows to introduce the human uncertain behavior to the computer is definite performance [33]. Tanaka & Sugeno [34] present that fuzzy set has made considerable progress in intelligent computing research and practice applications now. The rapid development of fuzzy control is indivisible from the support of the fuzzy set theory. Fuzzy set theory provides not only new scientific logic and methods for information science and cognitive science but also an effective method for intelligent information processing technology [35]. Since then, fuzzy numbers have been extensively investigated by many researchers. Fuzzy numbers are a powerful tool for modeling uncertainty and for processing vague or subjective information in mathematical models. Their directions of development are diverse and have been applied to very varied practical problems, for instance, in fuzzy optimization, fuzzy transportation problems, and fuzzy differential equation [36, 37].

2.2.2 Membership functions

Zadeh [33] extends the notion of binary membership to accommodate various "degrees of membership" on the real continuous interval [0,1], where the endpoints of 0 and 1 conform to no membership and full membership in Fig. 5 (a). The indicator function does for crisp sets but where the infinite number of values in between the endpoints can represent various degrees of membership for an element X in some set on the universe. The sets on the universe X are termed by Zadeh as fuzzy sets. Fuzzy sets consist of a membership function that is illustrated in Fig. 5

(b). A key difference between crisp and fuzzy sets is their membership function; a crisp set has a unique membership function, whereas a fuzzy set can have an infinite number of membership functions to represent it. For fuzzy sets, uniqueness is sacrificed, but flexibility is gained because the membership function can be adjusted to maximize the utility for a particular application [38].

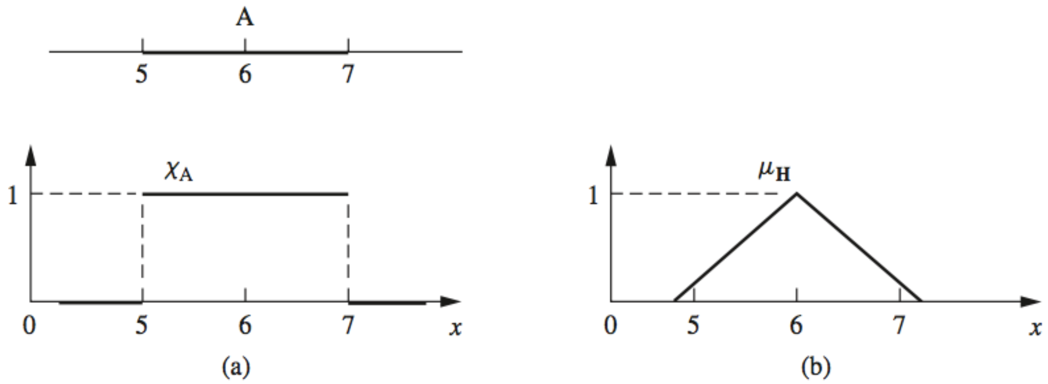

Fig. 5 Height membership functions for (a) a crisp set A, (b) a fuzzy set H.

The fuzzy logic build from several basic functions: The triangular function, Gaussian function, Trapezoidal function, Generalized bell function, Sigmoid function, π-Shaped function, Left–Right (LR) membership function, etc.

1. The simplest membership functions are formed using straight lines. Of these, the simplest is the triangular membership function, and it has the function name TRIMF. It is nothing more than a collection of three points forming a triangle. The graphical representation of the triangular membership function is shown in Fig. 6. [39]

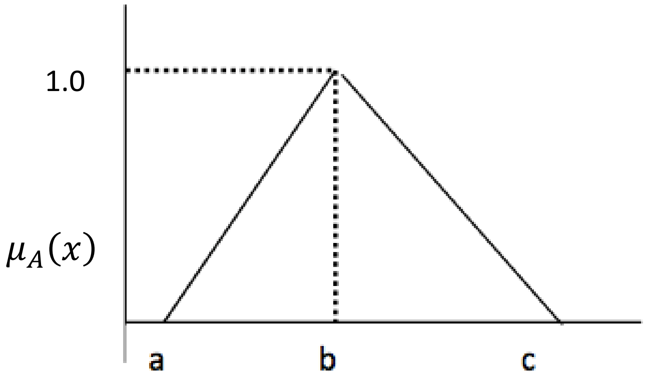

Fig. 6 Triangular membership function.
source: Mandal [39]

$$\mu(x) = \begin{cases} \frac{x-a}{b-a} & if\ a \le x \le b \\ \frac{c-x}{c-b} & if\ b \le x \le c \\ 0 & if\ x \le a\ or\ x \ge c \end{cases} \quad (2.6)$$

2. Trapezoidal membership function is defined by a lower limit a, an upper limit d, a lower support limit b, and an upper support limit c, where $a < b < c < d$ [40].

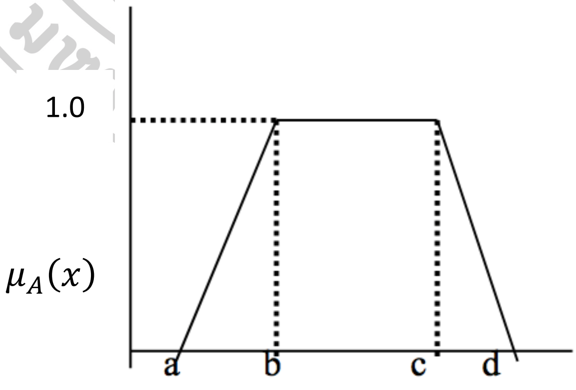

Fig. 7 Trapezoidal membership function.

$$\mu(x) = \begin{cases} 0, & (x < a)\ or\ (x > d) \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \end{cases} \tag{2.7}$$

3. The Gaussian membership function is usually represented as Gaussian(x: c, s) where c, s represents the mean and standard deviation [40].

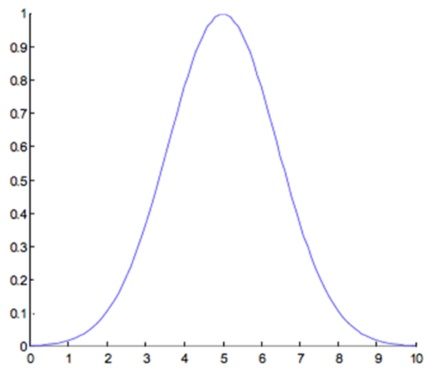

Fig. 8 Gaussian membership function.

$$\mu(x, c, s, m) = exp\left[-\frac{1}{2}\left|\frac{x-c}{s}\right|^{m}\right] \tag{2.8}$$

4. Generalized Bell membership function has three parameters: a-responsible for its width, c-responsible for its center, and b-responsible for its slopes as shown below [39].

$$\text{gbell}(\text{x}; \text{a}, \text{b}, \text{c},) = \frac{1}{1 + \left|\frac{x-c}{b}\right|^{2b}} \tag{2.9}$$

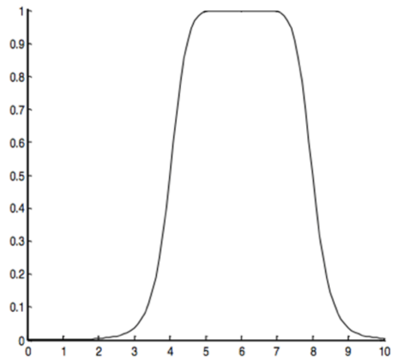

Fig. 9 Generalized Bell membership function.

5. A sigmoidal membership function has two parameters: a responsibility for its slope at the crossover point x = c. The membership function of the sigmoid function can be represented as Sig (x: a, c) as shown below [39].

$$\mathrm{Sig}(x; a, b, c,) = \frac{1}{1 + e^{-a(x-c)}} \quad (2.10)$$

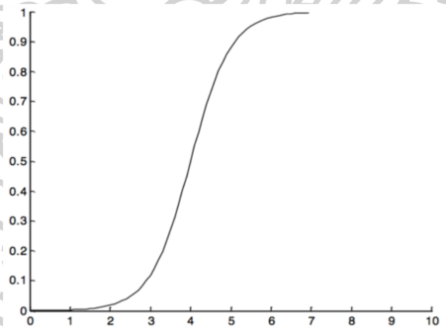

Fig. 10 Sigmoidal membership function.

6. Left–Right membership function or L-R membership function [41] is specified by three parameters {α, β, c}:

$$LR(x; c, \alpha, \beta) = \begin{cases} F_L\left(\frac{c-x}{\alpha}\right), x \le c \\ F_R\left(\frac{x-c}{\beta}\right), x \ge c \end{cases} \quad (2.11)$$

Where function $L(x)$ and function $R(x)$ are monotonically decreasing functions defined $[0, \infty)$ with $F_L(0) = F_R(0) = 1$ and $\lim_{x\to\infty} F_L(x) = \lim_{x\to\infty} F_R(x) = 0$.

$$F_L(x) = \max\left(0, \sqrt{1 - x^2}\right) \quad (2.12)$$

$$F_R(x) = e^{-|x|^3} \quad (2.13)$$

Based on the preceding $F_L(x)$ and $F_R(x)$, Fig.11 illustrates two L-R membership functions are specified by $LR\ (x; 65, 60, 10)$ and $LR\ (x; 25, 10, 40)$.

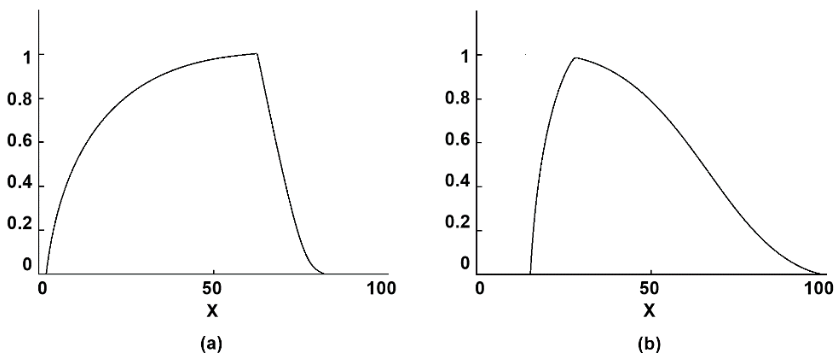

Fig. 11 Left –Right (LR) membership function.
source: Radhika & Parvathi, [41]

2.3 Fuzzy Analytic Hierarchy Process

2.3.1 Motivation

Fuzzy AHP is a synthetic extension of the classic AHP method when the fuzziness of the decision-makers is considered. The experts study the fuzzy AHP which is the extension of Saaty’s theory, providing evidence that fuzzy AHP shows a relatively more sufficient description of this kind of decision-making processes compare to the traditional AHP methods. The proposal of Cheng [42] presents a new algorithm for evaluating naval tactical missile systems by using the fuzzy AHP based on the grade value of the membership function. The assessment of Weck et al. [43] presents an alternative production cycle using fuzzy AHP. The discussion of Zhu et al. [44] presents the extent analysis method and applications of fuzzy AHP. The integration of Kuo et al. [45] presents fuzzy AHP and artificial neural networks for selecting convenience store locations. The employment of Yu [46] presents the characteristics of goal programming to solve group decision-making fuzzy AHP problem. The presentation of Sheu [47] presents a fuzzy-based approach to identifying global logistics strategies. The method of Kulak and Kahraman [48] presents the using fuzzy AHP for multi-criteria selection among transportation companies.

Consequently, the Fuzzy-AHP approach provides to eliminate the unnecessary criterion or criteria if all of the decision-makers assign an "absolutely not significant" value when compared with the other criteria and expresses the more significant criteria. Some experts does not accept this result whereas some think it is natural. Due to the fact that European culture is affected by the Aristo logic base on existence-nonexistence, which is called 0-1 logic, some European experts deny the fuzzy set theory. However, Japanese scientists adapt to the fuzzy set theory, and they use fuzzy logic in many different areas such as the production of the microwave oven, washing machines, scanners, photograph machines, and refrigerators. Consequently, fuzzy sets and related methods are still conflicting in the literature; so, fuzzy AHP applications have some risk in deployment, but the conventional AHP still cannot reflect the human thinking style. Avoiding these risks on performance, the fuzzy AHP, a fuzzy extension of AHP, is developed to solve the hierarchical fuzzy problems [49].

The FAHP method present Triangular Fuzzy Numbers (TFN). It can be identified as triple $x = (a, b, c)$, where defines a membership function as [50],

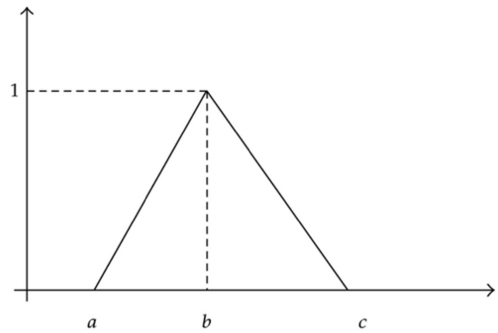

Fig. 12 Triangular fuzzy numbers
source: Chang, D.Y, [50]

$$\mu(x) = \begin{cases} \frac{x}{b-a} - \frac{a}{b-a}, & x\epsilon[a, b] \\ \frac{x}{b-c} - \frac{c}{b-c}, & x\epsilon[b, c] \\ 0 \quad , & oterwise \end{cases} \quad (2.14)$$

TFN is developed by AHP that is applied in order to compare a priority scale between each criterion as shown in Table 20.

Table 20 Linguistic terms and the corresponding TFN

Saaty scale	Definition	Fuzzy Triangular Scale
1	Equal importance	(1, 1, 1)
2	Intermediate values between the two adjacent judgments	(1, 2, 3)
3	Moderate importance	(2, 3, 4)
4	Intermediate values between the two adjacent judgments	(3, 4, 5)
5	Strong importance	(4, 5, 6)
6	Intermediate values between the two adjacent judgments	(5, 6, 7)
7	Very strongly importance	(6, 7, 8)

Saaty scale	Definition	Fuzzy Triangular Scale
8	Intermediate values between the two adjacent judgments	(7, 8, 9)
9	Extreme importance	(9, 9, 9)

The FAHP method present trapezoidal. The steps of the extent synthesis method are: Let $X = \{x_1, x_2, \ldots, x_n\}$ as an object set and $G = \{g_1, g_2, \ldots, g_m\}$ be a goal set. Each object is taken, and an extent analysis is performed for each goal. Consequently, m extent analysis values for each object can be obtained [24].

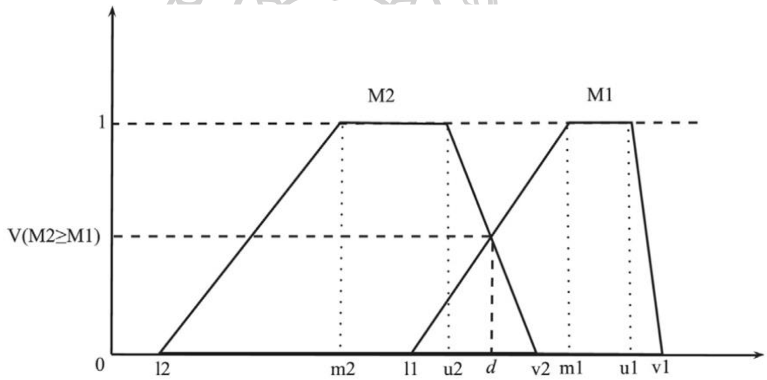

Fig. 13 Intersection of two trapezoidal fuzzy numbers
source: Sahin, [24]

The height of a fuzzy set $hgt(A)$ is the maximum of membership grades of A, $hgt(A) = \frac{sup}{\mathrm{x} \in \mathrm{X}} A(x)$

The degree of possibility of $M_2 = (l_2, m_2, u_2, v_2) \geq M_1 = (l_1, m_1, u_1, v_1)$ is defined as:

$$V(M_2 \geq M_1) = \frac{sup}{\mathrm{y} \geq \mathrm{x}} [\min\,(\mu M_1(x), \mu M_2(y))] \qquad (2.15)$$

$$V(M_2 \geq M_1) = \frac{sup}{y \geq x} [\min(\mu M_1(x), \mu M_2(y))]$$

and can be expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu M_2(d) \tag{2.16}$$

$$= \begin{cases} 1 & if\ \mu_2 \geq m_1 \\ \frac{v_2 - l_1}{(v_2 - \mu_2) + (m_1 - l_1)} & if\ \mu_2 \leq m_1, v_2 \geq l_1 \\ 0 & if\ v_2 \leq l_2 \end{cases} \tag{2.17}$$

Equation 2.17 illustrates that d is the y-axis value of the highest intersection point D between μM_1 and μM_2. Both $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$ should be known for the comparison of M_1 and M_2.

Table 21 Linguistic terms and corresponding trapezoidal fuzzy numbers

Definition	Fuzzy Trapezoidal Scale
Very low	(1, 1, 1, 2)
Low	(1, 2, 2, 3)
Middle low	(2, 3, 4, 5)
Middle	(4, 5, 5, 6)
Middle high	(5, 6, 7, 8)
High	(7, 8, 8, 9)
Very high	(8, 9, 9, 9)

The Gaussian fuzzy numbers is invented in order to overcome the shortcomings of triangular fuzzy numbers. It has a superiority over the preference scale results in real intersection between any number and all the other numbers. Gaussian fuzzy numbers treat equivalently and then the problem of getting some alternatives is removed by having the same rank [25].

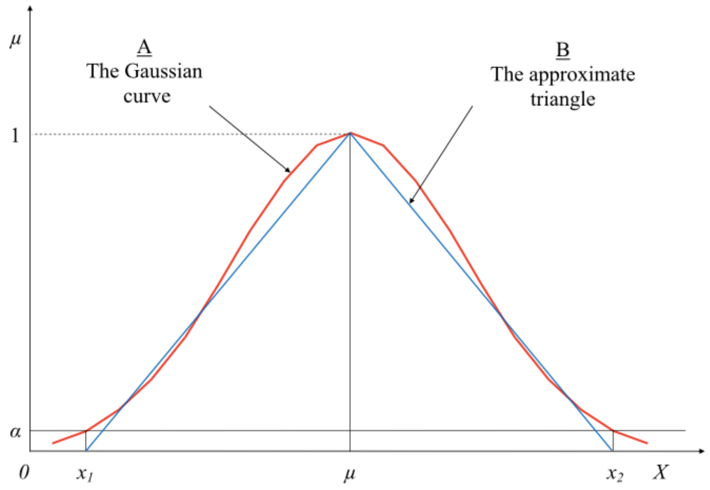

Fig. 14 Gaussian function A and it approximated triangle B.
source: Sahin & Yip, [25]

The steps of Gaussian fuzzy AHP are illustrated as given below: Let G_{ij} be the preference matrix after performing the triangular approximation, then:

$$S_i = \frac{\sum_j G_{ij}}{\sum_i \sum_j G_{ij}} = \frac{\sum_j (l_i^j, m_i^j, u_i^j)}{\sum_i \sum_j (l_i^j, m_i^j, u_i^j)} \tag{2.18}$$

where $l_i^j \cong m_i^j - \sigma_i^j \sqrt{-Ln(\alpha)}$ and $u_i^j \cong m_i^j + \sigma_i^j \sqrt{-Ln(\alpha)}$

σ level is set as 0.001 for triangular approximation.

$$S_i = \frac{(\sum_j l_i^j, \sum_j m_i^j, \sum_j u_i^j)}{(\sum_i \sum_j l_i^j, \sum_i \sum_j m_i^j, \sum_i \sum_j u_i^j)} = \left(\frac{\sum_j l_i^j}{\sum_i \sum_j u_i^j}, \frac{\sum_j m_i^j}{\sum_i \sum_j m_i^j}, \frac{\sum_j u_i^j}{\sum_i \sum_j l_i^j}\right) \tag{2.19}$$

$$\sum_j l_i^j = \sum_j m_i^j - \sum_j \sigma_i^j (\sqrt{-Ln(\alpha)}) \tag{2.20}$$

$$\sum_j u_i^j = \sum_j m_i^j + \sum_j \sigma_i^j (\sqrt{-Ln(\alpha)}) \tag{2.21}$$

$$\sum_i \sum_j l_i^j = \sum_i \sum_j m_i^j - \sum_i \sum_j \sigma_i^j (\sqrt{-Ln(\alpha)}) \quad (2.22)$$

$$\sum_i \sum_j u_i^j = \sum_i \sum_j m_i^j + \sum_i \sum_j \sigma_i^j (\sqrt{-Ln(\alpha)}) \quad (2.23)$$

where $m_{Si} = \frac{\sum_j m_i^j}{\sum_i \sum_j m_i^j}, X_{Si}^L = \frac{\sum_j l_i^j}{\sum_i \sum_j u_i^j}$ $and\ X_{Si}^R = \frac{\sum_j u_i^j}{\sum_i \sum_j l_i^j}$

Now, S_i is transformed into asymmetric Gaussian fuzzy number as follows:

$$\sigma_{Si}^L = \frac{m_{Si} - x_{Si}^L}{\sqrt{-Ln(\alpha)}} \quad (2.24)$$

$$\sigma_{Si}^R = \frac{x_{Si}^R - m_{Si}}{\sqrt{-Ln(\alpha)}} \quad (2.25)$$

where σ_{Si}^L illustrates the width of the left branch of the Gaussian fuzzy number and σ_{Si}^R expresses the width of the right branch of the Gaussian fuzzy number.

Table 22 Linguistic terms and the corresponding fuzzy Gaussian numbers ($\mu = Crisp\ number, \sigma = 0.5$)

Crisp number	Definition	Fuzzy Gaussian (μ, σ)
1	Equal importance	(1, 0.5)
2	Intermediate values between the two adjacent judgments	(2, 0.5)
3	Moderate importance	(3, 0.5)
4	Intermediate values between the two adjacent judgments	(4, 0.5)

5	Strong importance	(5, 0.5)
6	Intermediate values between the two adjacent judgments	(6, 0.5)
7	Very strongly importance	(7, 0.5)
8	Intermediate values between the two adjacent judgments	(8, 0.5)
9	Extreme importance	(9, 0.5)

source: Sahin & Yip, [25]

Membership function of an asymmetric Gaussian number is:

$$\mu_{Si}(x) = \begin{cases} exp\left[-(\frac{x - m_{Si}}{\sigma_{Si}^{L}})^2\right], & if\ x \leq m_{Si} \\ exp\left[-(\frac{x - m_{Si}}{\sigma_{Si}^{R}})^2\right], & if\ x > m_{Si} \end{cases} \quad (2.26)$$

The degree of possibility for a convex Gaussian fuzzy number S_i is greater than k convex Gaussian fuzzy number $S_i(i = 1, 2, \ldots, k)$ and can be defined by

$$V(S > S_1, S_2, \ldots, S_k) = V[(S > S_1), (S > S_2) \ldots (S > S_k)]$$
$$= \min_V (S > S_i),$$
$$i = 1, 2, 3, \ldots, k.$$

Assume that $d'(A_i) = \min_V (S_i > S_j)$ for j = 1, 2,..,n; j ≠ i. Then the weight vector is given by: $W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T$ where $A_i(i = 1, 2, \ldots, n)$ are n elements.

The normalized weight vectors via normalization are:

$$W = (d(A_1), (d(A_2), \ldots, (d(A_n))^T \quad (2.27)$$

where

$$d(A_i) = \frac{d'(A_i)}{\sum_i d'(A_i)} \tag{2.28}$$

2.3.2 Applications of fuzzy analytic hierarchy process

Sadeghi et al. [51] present a factors affecting high-tech SME's success should be measured not separately. They make use of Multi-Attribute Decision-Making (MADM) approach, which allows multi-criteria and simultaneous evaluation. Proposed model to achieve mentioned targets is composed of the following steps.

Step 1: Form a committee of experts

For the application, an expert team with 6 members is formed. These experts are university professors and managers of high-tech firms.

Step 2: Identify the factors and sub-factors to be used in the model

In this study, reviewing the literature and interviewing with experts, 13 intra-organizational and 34 inter-organizational success factors identify to be effective in high-tech SME's success. Also these factors categorize into 10 main criteria (organizational, product characteristics, entrepreneurs characteristics, human resource, strategic, policies and regulations, financial, firm expertise, technological and market characteristics).

Step 3: Structure the AHP model hierarchy

The AHP model hierarchy is structured using the factors and sub-factors identified at Step 2.

Step 4: Determine the local weights

Using pairwise comparison matrices to determine the local weights of the factors and sub-factors. The fuzzy scale regarding relative significance to measure the relative weights as shown in Table 23.

Table 23 The linguistic scale for relative dominance and their corresponding triangular fuzzy numbers

Definition	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Just equal	(1, 1, 1)	(1, 1, 1)
Equal dominance	(1/2, 1, 3/2)	(2/3, 1, 2)
Weak dominance	(1, 3/2, 2)	(1/2, 2/3, 1)
Strong dominance	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strong dominance	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolute dominance	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

source: Sadeghi et al, [51]

Step 5: Comparison of criteria

Table 24 compute the global weights for the sub-factors. Global sub-factor weights are computed by multiplying local weight of the sub-factor with the local weight of the factor as shown in Table 25.

Table 24 Comparison of main criteria

	$Criteria_1$	$Criteria_2$	...	$Criteria_n$
$Criteria_1$	(1, 1, 1)	(3/2, 2, 5/2)	...	(X_1, X_2, X_3)
$Criteria_2$:	(2/5, 1/2, 2/3) :	(1, 1, 1) :	...	$(1/Y_1, 1/Y_2, 1/Y_3)$:
$Criteria_n$	$(1/X_1, 1/X_2, 1/X_3)$	(Y_1, Y_2, Y_3)	...	(1, 1, 1)

Table 25 Fuzzy AHP analysis results.

Main Criteria	Factors	Local Weight	Global Weight
Human resource (0.115)	Expertise and competence	0.32	0.037
	Experience	0.25	0.029
	Education	0.17	0.02
	Teamwork skills	0.25	0.029
Strategic (0.111)	Strategic planning	0.35	0.039
	Flexibility	0.31	0.034
	Reengineering	0.22	0.024
	Strategic Alliance	0.13	0.014
Technological (0.093)	Access to skilled workforce	0.40	0.037
	Ability to import equipment	0.30	0.028
	Relation between industry and university	0.30	0.028
Financial (0.110)	The initial Investment	0.37	0.04
	Liquidity	0.31	0.034
	Firms access to financial resources	0.33	0.036
Entrepreneurs characteristics (0.112)	Experience	0.195	0.022
	Risk Taking	0.147	0.017
	Creativity and innovation	0.147	0.017
	Leadership skills	0.196	0.022
	Managerial style	0.196	0.022
	Family support	0.12	0.013
Organizational (0.081)	Organizational structure	0.13	0.011
	Organizational culture	0.17	0.014
	Firm Life Cycle	0.21	0.017
	Being a learning organization	0.22	0.018
	size	0.09	0.007

Main Criteria	Factors	Local Weight	Global Weight
	up-to-dateness	0.19	0.015
Product characteristics (0.116)	Product Price	0.166	0.019
	Product quality	0.199	0.023
	Uniqueness of product	0.189	0.022
	After sales service	0.161	0.019
	Easiness of use	0.127	0.015
	Product Life cycle	0.158	0.018
Firm expertise (0.078)	Marketing	0.22	0.017
	Human resource management	0.14	0.011
	Finance & accounting	0.15	0.012
	R$D	0.26	0.021
	Customer Service	0.22	0.018
policies and regulations (0.082)	Relationship with global market Government support	0.08	0.007
	Copyright and Intellectual	0.25	0.021
	Property Rights	0.25	0.021
	SMEs protection laws	0.24	0.020
	Labor laws	0.17	0.014
Market characteristics (0.099)	Demand	0.30	0.03
	Intensity of competition in the industry	0.28	0.028
	Degree of uncertainty in the industry	0.20	0.02
	Access to suppliers	0.14	0.013
	Access to distribution channels	0.08	0.007

source: Sadeghi et al, [51]

The factor weights can be found using fuzzy AHP, it can be determined which factors has more effect on SME's success. The three important main factors in SME's success are Human resource, Product characteristics and Entrepreneurs characteristics. The results of this study also suggest that Strategic planning, Initial investment and access to skilled workforce are the most important sub-factors for high-tech SME's success.

According to the study by Sadeghi et al. [51], it can be seen that there are 10 main criteria, resulting in a pairwise comparison of up to 45 pairs as follows; Human resource has 4 secondary criteria which requires a pairwise comparison of up to 6 pairs, Strategic has 4 secondary criteria which requires a pairwise comparison of up to 6 pairs, Technological has 3 secondary criteria which requires a pairwise comparison of up to 3 pairs, Financial has 3 secondary criteria which requires a pairwise comparison of up to 3 pairs, Entrepreneurs characteristics has 6 secondary criteria which requires a pairwise comparison of up to 15 pairs, Organizational has 6 secondary criteria which requires a pairwise comparison of up to 15 pairs, Product characteristics has 6 secondary criteria which requires a pairwise comparison of up to 15 pairs, Firm expertise has 5 secondary criteria which requires a pairwise comparison of up to 10 pairs, Policies and Regulations has 5 secondary criteria which requires a pairwise comparison of up to 10 pairs, Market characteristics has 5 secondary criteria which requires a pairwise comparison of up to 10 pairs. In total, there are up to 138 pairs of comparisons which may cause experts to mistake while performing comparisons.

2.4 Related Works on AHP and FAHP

2.4.1 Related works on AHP

AHP is invented in the 1980s, it is being used continuously until today to help make decisions in various types of tasks for efficiency and continuous development as shown in Table 26.

Table 26 References on the topic of AHP

Authors	Research Article	Application Areas	Objective	Scale	Comparison (times)
Abastante et al. [52]	A new parsimonious AHP methodology: Assigning priorities to many objects by comparing pairwise few reference objects	Education	B	3, 4, 5	19
Fu. [53]	An integrated approach to catering supplier selection using AHP-ARAS-MCGP methodology	Airline Industry	A	5	10
Benmoussa et al. [54]	AHP-based Approach for Evaluating Ergonomic Criteria	Ergonomics	A, B	16	120
Ghimire et al. [55]	An analysis on barriers to renewable energy development in the context of Nepal using AHP	Energy	B	6, 5, 4, 3, 2	35

Authors	Research Article	Application Areas	Objective	Scale	Comparison (times)
Ozdemir & Sahin. [56]	Multi-criteria decision-making in the location selection for a solar PV power plant using AHP	Energy	A	5, 3	13
Promentilla et al. [57]	Teaching Analytic Hierarchy Process (AHP) in undergraduate chemical engineering courses	Education	A	5	10
Nayak et al. [16]	Deadline sensitive lease scheduling in cloud computing environment using AHP.	Business	A	3	3
Durmusoglu. [58]	Assessment of techno-entrepreneurship projects by using Analytical Hierarchy Process (AHP)	Technology	A	2, 4, 5	17
Chaouachi et al. [59]	Multi-criteria selection of offshore wind farms: Case study for the Baltic States	Energy	A	6	15

Authors	Research Article	Application Areas	Objective	Scale	Comparison (times)
Hillerman et al. [60]	Applying clustering and AHP methods for evaluating suspect healthcare claims	Healthcare	B	3	3
Breaz et al. [32]	Selecting industrial robots for milling applications using AHP	Industry	A	3, 7	24
Phudphad et al. [61]	Rankings of the security factors of human resources information system (HRIS) influencing the open climate of work: using analytic hierarchy process (AHP)	Human Resources	B	3, 5	13
Aşchilean et al. [31]	Choosing the optimal technology to rehabilitate the pipes in water distribution systems using the AHP method	Water Distribution Network	A	7	21
Dong & Cooper. [62]	An orders-of-magnitude AHP supply chain risk assessment framework	Industry	B	5	10

Authors	Research Article	Application Areas	Objective	Scale	Comparison (times)
Garbuzova-Schlifter et al. [20]	AHP-based risk analysis of energy performance contracting projects in Russia	Energy	B	8, 4, 3	37
Morano et al. [63]	Cultural heritage valorization: an application of AHP for the choice of the highest and best use	Investment	A	8, 5	38
Lee et al. [64]	Deriving Strategic Priority of Policies for Creative Tourism Industry in Korea using AHP	Tourism	B	4	6
Brudermann et al. [65]	Agricultural biogas plants – A systematic analysis of strengths, weaknesses, opportunities and threats	Energy	B	4	6
Sadeghi & Ameli. [66]	An AHP decision making model for optimal allocation of energy subsidy among socio-economic subsectors in Iran	Energy	A	6	15

A = Selection, B = Priority

From Table 26, it shows that during 2012-2019, AHP has been brought in multi-decision decision-making continuously in various tasks such as energy, education, ergonomics, business, healthcare, tourism, technology, and industry. The objective is to decide the target and compare the weight of the importance. It also shows the criteria that used to make decisions, most of which use a small scale. Due to the large scale, the size of the comparison may increase as a result in a consistency ratio. Benmoussa et al. [54], however, present 4 criteria for ergonomics. This research introduces the sub-criteria to determine the weight of importance, resulting in scale = 16, Due to the factors of ergonomics, there are many important criteria in human work analysis. If combine the comparison in total, it will be 120-time comparisons needed that the experts need to undergo which is a hard complex.

2.4.2 Related works on FAHP

FAHP is a combination of Fuzzy set with AHP which increases the consistency ratio to be more accurate [67]. Which is used to compare the pair. FAHP is a combination of many membership functions as shown in Table 27.

Table 27 References on the topic of ‘FAHP’

Authors	Research Article	Application Areas	Objective	Membership Functions
Debnath et al. [68]	Air quality assessment using weighted interval type-2 fuzzy inference system.	Ecological	B	I

Authors	Research Article	Application Areas	Objective	Membership Functions
Yadegaridehkordi et al. [21]	Predicting the adoption of cloud-based technology using fuzzy analytic hierarchy process and structural equation modelling approaches	Technology Computer	B	II
Khoshi et al. [69]	The data on the effective qualifications of teachers in medical sciences: An application of combined fuzzy AHP and fuzzy TOPSIS methods	Education	B	II
Alam et al. [70]	An Uncertainty-aware Integrated Fuzzy AHP-WASPAS Model to Evaluate Public Cloud Computing Services	Computer	B	II
Ligus & Peternek. [71]	Determination of most suitable low-emission energy technologies development in Poland using integrated fuzzy AHP-TOPSIS method	Technology, Energy	B	II
Ooi et al. [72]	Integration of Fuzzy Analytic Hierarchy Process into multi-objective Computer Aided Molecular Design	Computer, Chemical Engineering	B	II

Authors	Research Article	Application Areas	Objective	Membership Functions
Zarghami et al. [73]	Customizing well-known sustainability assessment tools for Iranian residential buildings using Fuzzy Analytic Hierarchy Process	Residential Buildings	B	II
Sahin. [24]	Consistency control and expert consistency prioritization for FFTA by using extent analysis method of trapezoidal FAHP	Computer	B	I
Modak et al. [22]	Performance evaluation of outsourcing decision using a BSC and Fuzzy AHP approach: A case of the Indian coal mining organization	Energy	B	II
Sahin & Yip. [25]	Shipping technology selection for dynamic capability based on improved Gaussian fuzzy AHP model	Technology	A	III
Wichapa & Khokhajaikiat. [74]	Solving multi-objective facility location problem using the fuzzy analytical hierarchy process and goal programming: a case study on infectious waste disposal centers.	Healthcare	A	II

Authors	Research Article	Application Areas	Objective	Membership Functions
Le et al. [75]	Application of fuzzy-analytic hierarchy process algorithm and fuzzy load profile for load shedding in power systems	Energy	A	II
Radziszewska-Zielina et al. [19]	Supporting Partnering Relation Management in the Implementation of Construction Projects Using AHP and Fuzzy AHP Methods.	Construction project	A	II
Jakiel et al. [76]	FAHP model used for assessment of highway RC bridge structural and technological arrangements	Buildings	B	II
Ahmadi et al. [23]	An FCM–FAHP approach for managing readiness-relevant activities for ERP implementation.	Industry	B	II
Anojkumar et al. [77]	Comparative analysis of MCDM methods for pipe material selection in sugar industry	Industry	A	II
Kabir & Sumi. [78]	Power substation location selection using fuzzy analytic hierarchy process and PROMETHEE: A case study from Bangladesh	Energy	A	II

Authors	Research Article	Application Areas	Objective	Membership Functions
Shaverdi et al. [79]	Developing sustainable SCM evaluation model using fuzzy AHP in publishing industry	Industry	A	II
Ayhan. [80]	A FUZZY AHP approach for supplier selection problem: A case study in a gear motor company	Industry	A	II
Sadeghi et al. [51]	Developing a fuzzy group AHP model for prioritizing the factors affecting success of High-Tech SME's in Iran: A case study	Small and Medium Enterprises	B	II
Çelen et al. [81]	Performance assessment of Turkish electricity distribution utilities: An application of combined FAHP/TOPSIS/DEA methodology to incorporate quality of service	Electricity	B	II
Jia et al. [82]	The low carbon development (LCD) levels' evaluation of the world's 47 countries (areas) by combining the FAHP with the TOPSIS method	Energy	B	II
Chen & Yang. [67]	An MAGDM based on constrained FAHP and FTOPSIS and its application to supplier selection	Industry	A	II

Authors	Research Article	Application Areas	Objective	Membership Functions
Rostamzadeh et al. [83]	Prioritizing effective 7Ms to improve production systems performance using fuzzy AHP and fuzzy TOPSIS (case study)	Industry	B	II

A = Selection, B = Priority, I = Trapezoidal, II= Triangular, III = Gaussian

Table 27 shows that during 2011-2018, FAHP has been brought in multi-decision-making continuously in various tasks such as ecological energy, education, computer, buildings, SMEs, healthcare, technology, and industry. The objective is to decide the target and compare the weight of the importance. It shows that most FAHPs use a triangular membership function, which is less complicated than other functions.

During 2017-2018, it starts using the gaussian FAHP in technology and using trapezoidal FAHP in computer and ecological work, which are more complicated than triangular. Sahin & Yip [25] presents that the expert consistency prioritization is conducted for expertise differences instead of assuming experts identical or assigning some predefined weights. Gaussian AHP is that it produces more accurate and realistic results than the conventional FAHP methods. The trapezoidal FAHP is often used to find priority rather than selection. Sahin [24] also presents that the expert consistency prioritization is also implemented for FFTA by using the extent analysis method of trapezoidal FAHP. An analytic comparison between with and without consistency control is obtained. The numerical results for the collapse of an offshore platform are presented to illustrate the applicability of the approach.

2.5 Other Application Tools

2.5.1 Other application tools

From the past to the present, there are various developed tools to support the decision-making. It is reliable and accurate in decision-making with many tools to apply to various types of tasks as shown in Table 28.

Table 28 Other Application Tools

Other tools	Authors	Research Article	Application Areas
SAW, TOPSIS and fuzzy TOPSIS	Seyedmohammadi et al. [84]	Application of SAW, TOPSIS and fuzzy TOPSIS models in cultivation priority planning for maize, rapeseed and soybean crops	Agricultural
TOPSIS	Cambazoğlu et al. [85]	Geothermal resource assessment of the Gediz Graben utilizing TOPSIS methodology	Energy
TOPSIS	Luan et al. [86]	Evaluating Green Stormwater Infrastructure strategies efficiencies in a rapidly urbanizing catchment using SWMM-based TOPSIS	Storm water

Other tools	Authors	Research Article	Application Areas
TOPSIS	Ouenniche et al. [87]	An out-of-sample framework for TOPSIS-based classifiers with application in bankruptcy prediction	investment
TOPSIS, VIKOR	Baccour. [88]	Amended fused TOPSIS-VIKOR for classification (ATOVIC) applied to some UCI data sets	Healthcare
VIKOR	Chen. [89]	Remoteness index-based Pythagorean fuzzy VIKOR methods with a generalized distance measure for multiple criteria decision analysis	Internet
VIKOR	San Cristóbal. [90]	Multi-criteria decision-making in the selection of a renewable energy project in spain: The Vikor method	Energy
DCE	De Bekker-Grob et al. [91]	Are Healthcare Choices Predictable The Impact of Discrete Choice Experiment Designs and Models.	Healthcare

Other tools	Authors	Research Article	Application Areas
SAW and TOPSIS	Chen. [92]	Comparative analysis of SAW and TOPSIS based on interval-valued fuzzy sets: Discussions on score functions and weight constraints	Education
DCE	Meginnis et al. [93]	Strategic bias in discrete choice experiments	Energy
PROMETHEE	Zindani & Kumar. [94]	Material Selection for Turbine Seal Strips using PROMETHEE-GAIA Method	Material
PROMETHEE	Lopes et al. [95]	Regional tourism competitiveness using the PROMETHEE approach	Tourism
ELECTRE	Yu et al. [96]	ELECTRE methods in prioritized MCDM environment	Information
ELECTRE, TOPSIS	Micale et al. [97]	A combined interval-valued ELECTRE TRI and TOPSIS approach for solving the storage location assignment problem	Warehouse

Other tools	Authors	Research Article	Application Areas
SMART	Schader et al. [98]	Accounting for uncertainty in multi-criteria sustainability assessments at the farm level: Improving the robustness of the SMART-Farm Tool	Farm
SMART	Barfod et al. [99]	COPE-SMARTER - A decision support system for analysing the challenges, opportunities and policy initiatives: A case study of electric commercial vehicles market diffusion in Denmark	Electric Vehicles
ANP	Liang et al. [100]	Using the analytic network process (ANP) to determine method of waste energy recovery from engine	Energy

Other tools	Authors	Research Article	Application Areas
ANP	Chemweno et al. [101]	Development of a risk assessment selection methodology for asset maintenance decision making: An analytic network process(ANP) approach	Maintenance
ANP	Atmaca & Basar. [102]	Evaluation of power plants in Turkey using Analytic Network Process (ANP)	Energy

ANP = Analytic Network Process

DCE = Discrete Choice Experiment

SAW = Simple Additive Weighting

TOPSIS = Technique for Order Preference by Similarity to the Ideal Solution

PROMETHEE = Preference Ranking Organization METHod for Enrichment of Evaluations

VIKOR = Vlse Kriterijumska Optimizacija Kompromisno Resenje

SMART = Simple Multi Attribute Rating Technique

ELECTRE = ELimination Et Choix Traduisant la REalite

Table 28 has shown that the multi-criteria decision analysis and decision tool have many forms, but the current popularity of AHP is TOPSIS, VIKOR, PROMETHEE, ELECTRE, SMART, and ANP.

The second most popular tool after AHP is TOPSIS. It was initially developed by Hwang and Yoon in 1981 [4]. TOPSIS is a procedure methodology consisting of the forming of the decision matrix, followed by decision matrix normalization and weighted normalized decision matrix. Then, a step of computing the positive and negative ideal solutions and determining separation measures for each alternative is to be done. The last step is committed to calculate the relative closeness coefficients and ranking the alternatives in descending order based on the corresponding values of closeness coefficients [103].

The VIKOR method uses linear normalization, and the normalized values do not depend on the evaluation unit of a criterion. It is an aggregating function representing the distance from the ideal solution, considering the relative importance of all criteria and balancing total and individual satisfaction [104].

PROMETHEE is developed by Brans (1982); there are several absorbing applications of PROMETHEE, such as management, logistics, financial, or tourism applications. In the PROMETHEE method, actions first compare a pair of criteria according to decision-maker preferences, resulting in local scores. These local scores are then aggregated to global scores, obtaining a partial pre-order rank, PROMETHEE I, or a complete pre-order rank, PROMETHEE II [95].

The ANP, also invented by Saaty in 1996, is a generalization of the AHP. While the AHP represents a framework with a unidirectional hierarchical AHP relationship, the ANP allows for complex interrelationships among decision levels and attributes. The ANP feedback approach replaces hierarchies with networks in which the relations between levels are not easily represented as higher or lower, dominant or subordinate, direct or indirect. For instance, not only does the significance of the criteria determine the significance of the alternatives in a hierarchy but also the significance of the alternatives may impact the importance of the criteria. Consequently, a hierarchical representation with a linear top-to-bottom structure is unsuitable for a complex system [100].

Simple Multi Attribute Rating (SMART) is an extensive decision-making model that accounts for quantitative and qualitative things. In a decision-making model, SMART attempts to cover any shortage from the previous model without computerization. The SMART weighting method is a method of supporting the most straightforward decision. In this method seen, some parameters determine the decision. These parameters have a range of weights and values. The weighting of SMART using a scale between 0 and 1. The value will be the factor of the decision taken. Consequently, simplifies the comparison and calculation of the value of each alternative [105].

ELECTRE is one of the multiple-criteria decision-making method based on the notion of outranking using a pairwise comparison of the alternatives based on any suitable criteria. ELECTRE method is used under conditions where the alternative is less by the criteria reasonable and eliminated alternative could be generated; in other words, ELECTRE is used for cases with many options. but only a few criteria involved [106].

2.5.2 Hybrid AHP tools

From the past to the present, AHP has been applied in various methods to analyze data sets to be suitable for different types of tasks and maximize benefits, as shown in Table 29.

Table 29 AHP & Other Application Tools

AHP & Other tools	Authors	Research Article	Application Areas
SWOT-AHP-TOWS	Gottfried et al. [107]	SWOT-AHP-TOWS analysis of private investment behavior in the Chinese biogas sector	Energy
FAHP-DEA	Otay et al. [108]	Multi-expert performance evaluation of healthcare institutions using an integrated intuitionistic fuzzy AHP&DEA methodology	Healthcare
AHP-TOPSIS- GIS	Akgün et al. [109]	Solving an ammunition distribution network design problem using multi- objective mathematical modeling, combined AHP-TOPSIS, and GIS	Military
AHP-DEA	Wang et al. [110]	An integrated AHP–DEA methodology for bridge risk assessment	Buildings
SWOT-AHP	Szulecka et al. [111]	Forest plantations in Paraguay: Historical developments and a critical diagnosis in a SWOT-AHP framework	Ecological

AHP & Other tools	Authors	Research Article	Application Areas
AHP-TOPSIS	Wang et al. [112]	Application of AHP, TOPSIS, and TFNs to plant selection for phytoremediation of petroleum-contaminated soils in shale gas and oil fields	Energy
SWOT-AHP Fuzzy-TOPSIS	Solangi et al. [113]	Evaluating the strategies for sustainable energy planning in Pakistan: An integrated SWOT-AHP and Fuzzy-TOPSIS approach	Energy
AHP-VIKOR-PROMETHEE	Sennaroglu et al. [114]	A military airport location selection by AHP integrated PROMETHEE and VIKOR methods	Location selection
Fuzzy- AHP-VIKOR- DEA	Suganthi. [115]	Multi expert and multi criteria evaluation of sectoral investments for T sustainable development: An integrated fuzzy AHP, VIKOR / DEA methodology	Smart cities
Fuzzy-AHP–VIKOR	Rezaie et al. [116]	Evaluating performance of Iranian cement firms using an integrated fuzzy AHP–VIKOR method	Industrial
AHP-VIKOR	Soner et al. [117]	Application of AHP and VIKOR methods under interval type 2 fuzzy environment in maritime transportation	Transportation

AHP & Other tools	Authors	Research Article	Application Areas
AHP-DCE	Danner et al. [118]	Comparing Analytic Hierarchy Process and Discrete-Choice Experiment to Elicit Patient Preferences for Treatment Characteristics in Age-Related Macular Degeneration	Healthcare
AHP–ELECTRE	Żak et al. [119]	Application of AHP and ELECTRE III/IV methods to multiple level, multiple criteria evaluation of urban transportation projects	Transportation
AHP-DCE	Marsh et al. [120]	Multiple Criteria Decision Analysis for Health Care Decision Making—Emerging Good Practices: Report 2 of the ISPOR MCDA Emerging Good Practices Task Force	Healthcare
AHP–ELECTRE	Kaya et al. [121]	An integrated fuzzy AHP–ELECTRE methodology for environmental impact assessment	Environment
AHP-Promethee	Kazan et al. [122]	Election of Deputy Candidates for Nomination with AHP-Promethee Methods	Election

Table 29 shows that the AHP is a popular decision-making tool for various types of tasks. The AHP is an essential tool for making difficult decisions more accessible, i.e., AHP and SWOT for applying AHP method to identify the weights for strengths, weaknesses, opportunities, and threats [113]. It is also used in conjunction with other tools, e.g., SWOT, TOWS, DEA, TOPSIS, VIKOR, etc.

CHAPTER 3

The Scaling score AHP for Large Criteria Decision-Making Problems

In this chapter, the researcher put the AHP technique into use in classifying and analyzing the factors and alternatives with multiple-criteria decision-making calculation to find out the best and most appropriate factor and alternative. Regarding the methodology, Classic Analytic Hierarchy Process is used in the computation, and suppose we have 10 main criteria. In the case of parallel comparison, the experts must conduct the comparison 45 times. For 10 selected alternatives of each criterion, the experts need to complete the comparison 450 times. If we combine the comparison in total, it will be 495-time comparisons needed that the experts need to undergo, which is a bit complex and hard to achieve. To solve this matter, we require the experts to grade the importance from 1 to 9 for criteria and alternatives for their convenience. The method is called "Normalize function-based scaling AHP".

3.1 Modified analytic hierarchy process

The assumptions of this research are essentially the same as those of researchers using the basic AHP model, except for a scaling score method for modified criteria decision-making problems. Let the criteria be a set of properties or attributes concerning which elements in the goal are compared. We will refer to the elements of criteria as C.

Let C be a criterion with $C_1, C_2, C_3, \dots\ C_n$

C_n be a scaling scoring with 1, 2, 3, 4, 5, 6, 7, 8, 9.

$$\begin{matrix} C_1 - C_1, & C_1 - C_2, & C_1 - C_3, & \ldots & , C_1 - C_n \\ C_2 - C_1, & C_2 - C_2, & C_2 - C_3, & \ldots & , C_2 - C_n, \\ C_3 - C_1, & C_3 - C_2, & C_3 - C_3, & \cdots & , C_3 - C_n, \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n - C_1, & C_n - C_2, & C_n - C_3, & \cdots & , C_n - C_n, \end{matrix}$$

when
$$C_{ij} = \begin{cases} C_i - C_j \geq 0, then\ C_{ij} = (C_i - C_j) + 1 \\ C_i - C_j < 0, then\ C_{ij} = \frac{1}{-[(C_i - C_j) - 1]} \end{cases} \quad (3.1)$$

The researcher propose assigning a significant scale from 1 to 9. The scaling scoring of alternative criteria is analyzed as shown in Table 30.

Table 30 Fundamental scale of Normalize function-based scaling AHP

Verbal Judgments	Intensity of Importance
Lowest	1
Weakly	3
Moderate	5
Very strongly	7
Extreme	9
Intermediate values between the two adjacent judgments	2, 4, 6, 8

Then the judgment, matrix A, which contains comparison value C_{ij} for all $i, j \in \{1, 2, \ldots, n\}$ is given by (3.2) [123].

$$A = \begin{bmatrix} C_{11} & C_{12} & C_{13} & \cdots & C_{1n} \\ \frac{1}{C_{21}} & C_{22} & C_{23} & \cdots & C_{2n} \\ \frac{1}{C_{31}} & \frac{1}{C_{32}} & C_{33} & \ldots & C_{3n} \\ \vdots & & \vdots & & \vdots \\ \frac{1}{C_{n1}} & \frac{1}{C_{n2}} & \frac{1}{C_{n3}} & \cdots & C_{nn} \end{bmatrix} \quad (3.2)$$

For multiple the decision-makers, let h be the number of decision makers and $C_{\mathrm{ij}}^{\mathrm{k}}$ be the comparison value of criteria i and j given by decision-maker k, where $k = 1, 2, \ldots, h$. Then by using geometric mean of the $C_{\mathrm{ij}}^{\mathrm{k}}$ conducted by each decision maker, we have a new judgment matrix with the elements given by (3.3) [123].

$$C_{ij} = (C_{ij}^1 * C_{ij}^2 * C_{ij}^3 * \ldots * C_{ij}^k * \ldots * C_{ij}^h)^{1/h} = (\Pi_{k=1}^h c_{ij}^k)^{1/h} \qquad (3.3)$$

3.1.1. Normalize each column to get a new judgment, matrix A.

$$A' = \begin{bmatrix} c'_{11} & c'_{12} & \cdots & c'_{1n} \\ c'_{21} & c'_{22} & & c'_{2n} \\ \vdots & & \ddots & \vdots \\ c'_{n1} & c'_{n2} & \cdots & c'_{nn} \end{bmatrix} = \begin{bmatrix} c_{11}/\sum_{i=1}^n c_{i1} & c_{12}/\sum_{i=1}^n c_{i2} & \cdots & c_{1n}/\sum_{i=1}^n c_{in} \\ c_{21}/\sum_{i=1}^n c_{i1} & c_{22}/\sum_{i=1}^n c_{i2} & & c_{2n}/\sum_{i=1}^n c_{in} \\ & \vdots & \ddots & \vdots \\ c_{n1}/\sum_{i=1}^n c_{i1} & c_{n2}/\sum_{i=1}^n c_{i2} & \cdots & c_{nn}/\sum_{i=1}^n c_{in} \end{bmatrix} \qquad (3.4)$$

where $\sum_{i=1}^n c_{ij}$ is the sum of column j of the judgment, matrix A.

3.1.2. Sum up each row of normalized judgment matrix A'to get weight vector V.

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^n c'_{1j} \\ \sum_{j=1}^n c'_{2j} \\ \vdots \\ \sum_{j=1}^n c'_{nj} \end{bmatrix} \qquad (3.5)$$

3.1.3. Define the final normalization weight vector W.

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} v_1/\sum_{i=1}^n v_i \\ v_2/\sum_{i=1}^n v_i \\ \vdots \\ v_n/\sum_{i=1}^n v_i \end{bmatrix} \qquad (3.6)$$

3.1.4. Check consistency

In the next step, we use the consistency, checking method developed by Thomas L. Saaty. He determined the Consistency Ratio (CR) in the following equations [32].

$$CI = \frac{(\lambda_{max} - n)}{n-1} \tag{3.7}$$

where CI = Consistency Index

n = Number of elements in the matrix

λ_{max} = The largest eigenvalue of a matrix

$$CR = \frac{CI}{RI} \tag{3.8}$$

CR = Consistency Ratio

RI = Random Index computed for matrices that depend on n, as shown in Table 31.

Table 31 Random index values

n	3	4	5	6	7	8	9	10
RI	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

source: Aşchilean et al. [31]

The decision is acceptable if the consistency ratio is less than or equal to 0.1. However, if it is not, the analyst must redo the whole process [30].

3.1.5. Result of the overall rating.

Finally, the criteria are ordered with the weights decreasingly. The most important criterion has the most significant weight. On the other hand, the least essential criterion has the most negligible weight.

3.2 Applications of analytic hierarchy process

In this study, the researchers put the AHP technique into use in classifying and analyzing the factors and alternatives in the decision-making of constructing a power station in line with multiple-criteria decision-making calculation so that we can find out the best and most appropriated factor and alternative with help stabilize the amount of electricity distributed. Regarding the methodology, Classic Analytic Hierarchy Process is used in the calculation, and there are 7 main criteria. In the case of parallel comparison, the experts must conduct the comparison 21 times. For 10 elected alternatives of each criterion, the experts need to complete the comparison 315 times. If we combine the comparison in total, it will be 336-time comparisons needed that the experts need to undergo, which is a bit complex and hard to achieve. To solve this matter, we require the experts to grade the importance from 1 to 9 for 7 criteria and rank 10 alternatives power station construction projects for their convenience. There are 7 criteria we used to assess data for the power station construction project: Electricity income (C_1), Electrical consumption (C_2), The problem by the system (C_3), The number of electrical users (C_4), The forecast of power shortage (C_5), Establishment of electrical transmission lines (C_6), The acceptance for the community (C_7) and 10 power station construction projects. Accordingly, we applied our proposed method to solve such a large-scale decision problem. By deploying our technique, the decision-makers are needed to make only 77 times. This study investigation and analyzes the differences between the classic AHP and AHP for large scale.

3.2.1 Applications of the modified AHP

The experts must rate the score from 1-9 for each criterion. On the next step, we will collect the scoring and calculate using inequality (3.1-3.2), shown in Tables 32-33.

Table 32 Rating of each criterion for construction of power station

Criterion	9	8	7	6	5	4	3	2	1
C_1: Electricity income			/						
C_2: Electrical consumption	/								
C_3: The problem by system		/							
C_4: The number of electrical users				/					
C_5: The forecast of power shortage	/								
C_6: Establishment of electrical transmission lines						/			
C_7: The acceptance for community							/		

Table 33 Scoring and calculate

	$C_i - C_j \geq 0$	$C_i - C_j < 0$	C_{ij}
$C_1 - C_1$	$7-7=0 \geq 0$ *then* $C_{11}=(7-7)+1$	-	$C_{11}=1$
$C_1 - C_2$	-	$7-9=-2 < 0$, *then* $C_{12}=\frac{1}{-[(7-9)-1]}$	$C_{12}=\frac{1}{3}$
$C_1 - C_3$	-	$7-8=-1 < 0$, *then* $C_{13}=\frac{1}{-[(7-8)-1]}$	$C_{13}=\frac{1}{2}$
$C_1 - C_4$	$7-6=1 \geq 0$ *then* $C_{14}=(7-6)+1$	-	$C_{14}=2$
$C_1 - C_5$	-	$7-9=-2 < 0$, *then* $C_{15}=\frac{1}{-[(7-9)-1]}$	$C_{15}=\frac{1}{3}$
$C_1 - C_6$	$7-4=3 \geq 0$ *then* $C_{16}=(7-4)+1$	-	$C_{16}=4$

	$C_i - C_j \geq 0$	$C_i - C_j < 0$	C_{ij}
$C_1 - C_7$	$7 - 3 = 4 \geq 0$ *then* $C_{17} = (7 - 3) + 1$	-	$C_{17} = 5$
$C_2 - C_2$	$9 - 9 = 0 \geq 0$ *then* $C_{22} = (9 - 9) + 1$	-	$C_{22} = 1$
$C_2 - C_3$	$9 - 8 = 1 \geq 0$ *then* $C_{23} = (9 - 8) + 1$	-	$C_{23} = 2$
$C_2 - C_4$	$9 - 6 = 3 \geq 0$ *then* $C_{24} = (9 - 6) + 1$	-	$C_{24} = 4$
$C_2 - C_5$	$9 - 9 = 0 \geq 0$ *then* $C_{25} = (9 - 9) + 1$	-	$C_{25} = 1$
$C_2 - C_6$	$9 - 4 = 5 \geq 0$ *then* $C_{26} = (9 - 4) + 1$	-	$C_{26} = 6$
$C_2 - C_7$	$9 - 3 = 6 \geq 0$ *then* $C_{27} = (9 - 3) + 1$	-	$C_{27} = 7$
$C_3 - C_3$	$8 - 8 = 0 \geq 0$ *then* $C_{33} = (8 - 8) + 1$	-	$C_{33} = 1$
$C_3 - C_4$	$8 - 6 = 2 \geq 0$ *then* $C_{34} = (8 - 6) + 1$	-	$C_{34} = 3$
$C_3 - C_5$	-	$8 - 9 = -1 < 0, then$ $C_{35} = \frac{1}{-[(8-9)-1]}$	$C_{35} = \frac{1}{2}$
$C_3 - C_6$	$8 - 4 = 4 \geq 0$ *then* $C_{36} = (8 - 4) + 1$	-	$C_{36} = 5$
$C_3 - C_7$	$8 - 3 = 5 \geq 0$ *then* $C_{37} = (8 - 3) + 1$	-	$C_{37} = 6$
$C_4 - C_4$	$6 - 6 = 0 \geq 0$ *then* $C_{44} = (6 - 6) + 1$	-	$C_{44} = 1$
$C_4 - C_5$	-	$6 - 9 = -3 < 0, then$ $C_{45} = \frac{1}{-[(6-9)-1]}$	$C_{45} = \frac{1}{4}$
$C_4 - C_6$	$6 - 4 = 2 \geq 0$ *then* $C_{46} = (6 - 4) + 1$	-	$C_{46} = 3$
$C_4 - C_7$	$6 - 3 = 3 \geq 0$ *then* $C_{47} = (6 - 3) + 1$	-	$C_{47} = 4$
$C_5 - C_5$	$9 - 9 = 0 \geq 0$ *then* $C_{55} = (9 - 9) + 1$	-	$C_{55} = 1$
$C_5 - C_6$	$9 - 4 = 5 \geq 0$ *then* $C_{56} = (9 - 4) + 1$	-	$C_{56} = 6$
$C_5 - C_7$	$9 - 3 = 6 \geq 0$ *then* $C_{57} = (9 - 3) + 1$	-	$C_{57} = 7$
$C_6 - C_6$	$4 - 4 = 0 \geq 0$ *then* $C_{66} = (4 - 4) + 1$	-	$C_{66} = 1$
$C_6 - C_7$	$4 - 3 = 1 \geq 0$ *then* $C_{67} = (4 - 3) + 1$	-	$C_{67} = 2$

	$C_i - C_j \geq 0$	$C_i - C_j < 0$	C_{ij}
$C_7 - C_7$	$3 - 3 = 0 \geq 0$ *then* $C_{77} = (3 - 3) + 1$	-	$C_{77} = 1$

Then, find the weight and the consistency ratio value using inequality (3.3-3.8). The calculated values are shown in Table 34 below.

Table 34 Weight of importance and the consistency ratio value

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Weight (%)
C_1	1.00	0.33	0.50	2.00	0.33	4.00	5.00	11.83
C_2	3.00	1.00	2.00	4.00	1.00	6.00	7.00	27.74
C_3	2.00	0.50	1.00	3.00	0.50	5.00	6.00	17.80
C_4	0.50	0.25	0.33	1.00	0.25	3.00	4.00	8.04
C_5	3.00	1.00	2.00	4.00	1.00	6.00	7.00	27.74
C_6	0.25	0.17	0.20	0.33	0.17	1.00	2.00	4.00
C_7	0.20	0.14	0.17	0.25	0.14	0.50	1.00	2.85
CR	0.0242							

From Table 34, the criteria which are most concern are C_2 (Electrical consumption) and C_5 (The forecast of power shortage), with the weight of 27.74 %, followed by C_3 (The problem by the system) with the weight of 17.80 %. C_1 (Electricity income) criterion is ranked thirdly important, with the weight of a 11.83 %. The consistency ratio is 0.0242.

Table 35 Ranking of experts based on 7 criteria for potential power stations

Alternative \ Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Power Station A_1	9	8	5	7	9	5	5
Power Station A_2	7	7	5	6	9	5	5
Power Station A_3	5	5	7	4	9	5	5
Power Station A_4	1	1	6	2	8	5	5
Power Station A_5	2	2	5	3	9	5	5
Power Station A_6	3	3	9	5	9	5	5
Power Station A_7	4	4	5	1	9	5	5
Power Station A_8	9	9	8	9	9	5	5
Power Station A_9	9	9	8	9	8	5	5
Power Station A_{10}	6	6	5	8	9	5	5

From Table 35, experts rated their scores from 1-9 in each criterion compared with the alternatives to prioritize a power station construction. Then, we will calculate the weight and consistency ratio values using inequality (3.3-3.8). The results can be found in Tables 36-42.

Table 36 Weight of importance of alternatives and CR for C_1

C_1	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	3.00	5.00	9.00	8.00	7.00	6.00	1.00	1.00	4.00	21.55
A_2	0.33	1.00	3.00	7.00	6.00	5.00	4.00	0.33	0.33	2.00	11.06
A_3	0.20	0.33	1.00	5.00	4.00	3.00	2.00	0.20	0.20	0.50	5.72
A_4	0.11	0.14	0.20	1.00	0.50	0.33	0.25	0.11	0.11	0.17	1.55
A_5	0.13	0.17	0.25	2.00	1.00	0.50	0.33	0.13	0.13	0.20	2.08

C_1	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_6	0.14	0.20	0.33	3.00	2.00	1.00	0.50	0.14	0.14	0.25	2.89
A_7	0.17	0.25	0.50	4.00	3.00	2.00	1.00	0.17	0.17	0.33	4.07
A_8	1.00	3.00	5.00	9.00	8.00	7.00	6.00	1.00	1.00	4.00	21.55
A_9	1.00	3.00	5.00	9.00	8.00	7.00	6.00	1.00	1.00	4.00	21.55
A_{10}	0.25	0.50	2.00	6.00	5.00	4.00	3.00	0.25	0.25	1.00	7.98
CR	0.0347										

From Table 36, the most concerned criteria are alternatives 1, 8 and 9, with the weight of 21.55 % followed by alternative 2 with the weight of 11.06 %. Alternative 10 is ranked thirdly important, with the weight of 7.98 %. The consistency ratio of AHP is 0.0347.

Table 37 Weight of importance of alternatives and CR for C_2

C_2	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	2.00	4.00	8.00	7.00	6.00	5.00	0.50	0.50	3.00	16.32
A_2	0.50	1.00	3.00	7.00	6.00	5.00	4.00	0.33	0.33	2.00	11.60
A_3	0.25	0.33	1.00	5.00	4.00	3.00	2.00	0.20	0.20	0.50	5.90
A_4	0.13	0.14	0.20	1.00	0.50	0.33	0.25	0.11	0.11	0.17	1.58
A_5	0.14	0.17	0.25	2.00	1.00	0.50	0.33	0.13	0.13	0.20	2.12
A_6	0.17	0.20	0.33	3.00	2.00	1.00	0.50	0.14	0.14	0.25	2.96
A_7	0.20	0.25	0.50	4.00	3.00	2.00	1.00	0.17	0.17	0.33	4.18
A_8	2.00	3.00	5.00	9.00	8.00	7.00	6.00	1.00	1.00	4.00	23.53
A_9	2.00	3.00	5.00	9.00	8.00	7.00	6.00	1.00	1.00	4.00	23.53
A_{10}	0.33	0.50	2.00	6.00	5.00	4.00	3.00	0.25	0.25	1.00	8.28
CR	0.0346										

From Table 37, the most concerned criteria are alternatives 8 and 9, with the weight of 23.53 %, followed by alternative 1, with the weight of 16.32 %. Alternative 2 is ranked thirdly important, with the weight of 11.60 %. The consistency ratio of AHP is 0.0346.

Table 38 Weight of importance of alternatives and CR for C_3

C_3	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	1.00	0.33	0.50	1.00	0.20	1.00	0.25	0.25	1.00	4.20
A_2	1.00	1.00	0.33	0.50	1.00	0.20	1.00	0.25	0.25	1.00	4.20
A_3	3.00	3.00	1.00	2.00	3.00	0.33	3.00	0.50	0.50	3.00	11.37
A_4	2.00	2.00	0.50	1.00	2.00	0.25	2.00	0.33	0.33	2.00	7.28
A_5	1.00	1.00	0.33	0.50	1.00	0.20	1.00	0.25	0.25	1.00	4.20
A_6	5.00	5.00	3.00	4.00	5.00	1.00	5.00	2.00	2.00	5.00	25.77
A_7	1.00	1.00	0.33	0.50	1.00	0.20	1.00	0.25	0.25	1.00	4.20
A_8	4.00	4.00	2.00	3.00	4.00	0.50	4.00	1.00	1.00	4.00	17.28
A_9	4.00	4.00	2.00	3.00	4.00	0.50	4.00	1.00	1.00	4.00	17.28
A_{10}	1.00	1.00	0.33	0.50	1.00	0.20	1.00	0.25	0.25	1.00	4.20
CR	0.0072										

From Table 38, the most concerned criterion is alternative 6, with the weight of 25.77 %, followed by alternatives 8 and 9 with the weight of 17.28 %. Alternative 3 is ranked thirdly important, with the weight of 11.37 %. The consistency ratio of AHP is 0.0072.

Table 39 Weight of importance of alternatives and CR for C_4

C_4	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	2.00	4.00	6.00	5.00	3.00	7.00	0.33	0.33	0.50	11.60
A_2	0.50	1.00	3.00	5.00	4.00	2.00	6.00	0.25	0.25	0.33	8.28
A_3	0.25	0.33	1.00	3.00	2.00	0.50	4.00	0.17	0.17	0.20	4.18
A_4	0.17	0.20	0.33	1.00	0.50	0.25	2.00	0.13	0.13	0.14	2.12
A_5	0.20	0.25	0.50	2.00	1.00	0.33	3.00	0.14	0.14	0.17	2.96
A_6	0.33	0.50	2.00	4.00	3.00	1.00	5.00	0.20	0.20	0.25	5.90
A_7	0.14	0.17	0.25	0.50	0.33	0.20	1.00	0.11	0.11	0.13	1.58
A_8	3.00	4.00	6.00	8.00	7.00	5.00	9.00	1.00	1.00	2.00	23.53
A_9	3.00	4.00	6.00	8.00	7.00	5.00	9.00	1.00	1.00	2.00	23.53
A_{10}	2.00	3.00	5.00	7.00	6.00	4.00	8.00	0.50	0.50	1.00	16.32
CR	0.0346										

From Table 39, the most concerned criteria are alternatives 8 and 9 with the weight of 23.53 %, followed by alternative 10 with the weight of 16.32 %. Alternative 1 is ranked thirdly important, with the weight of 11.60 %. The consistency ratio of AHP is 0.0346.

Table 40 Weight of importance of alternatives and CR for C_5

C_5	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	2.00	1.00	11.11
A_2	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	2.00	1.00	11.11
A_3	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	2.00	1.00	11.11
A_4	0.50	0.50	0.50	1.00	0.50	0.50	0.50	0.50	1.00	0.50	5.56
A_5	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	2.00	1.00	11.11
A_6	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	2.00	1.00	11.11

C_5	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_7	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	2.00	1.00	11.11
A_8	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	2.00	1.00	11.11
A_9	0.50	0.50	0.50	1.00	0.50	0.50	0.50	0.50	1.00	0.50	5.56
A_{10}	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	2.00	1.00	11.11
CR	0.0000										

From Table 40, the most concerned criteria are alternatives 1, 2, 3, 5, 6, 7, 8 and 10, with the weight of 11.11 %, followed by alternatives 4 and 9, with the weight of 5.56 %. The consistency ratio of AHP is 0.00.

Table 41 Weight of importance of alternatives and CR for C_6

C_6	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_{10}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
CR	0.0000										

From Table 41, the most concerned criteria are alternatives 1-10, with the weight of 10.00 %. The consistency ratio of AHP is 0.00.

Table 42 Weight of importance of alternatives and CR for C_7

C_7	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
A_{10}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	10.00
CR	0.0000										

From Table 42, the most concerned criteria are alternatives 1-10, with the weight of 10.00 %. The consistency ratio of AHP is 0.00.

Obtaining such results from Tables 36-42, it is now possible to generate matrix A_{ij}^{C}. The columns in matrix C are put into order in the order of the criteria determined in Table 34; we found w^T = [11.83 27.74 17.80 8.04 27.74 4.00 2.85]. Performing the multiplication of the matrix and the vector weight, the preference vector for the ten power station construction project appears according to the following relation:

$$x = A_{ij}^{C} \times w^{T} = \begin{bmatrix} 21.55 & 16.32 & 4.20 & 11.60 & 11.11 & 10.00 & 10.00 \\ 11.06 & 11.60 & 4.20 & 8.28 & 11.11 & 10.00 & 10.00 \\ 5.72 & 5.90 & 11.37 & 4.18 & 11.11 & 10.00 & 10.00 \\ 1.55 & 1.58 & 7.28 & 2.12 & 5.56 & 10.00 & 10.00 \\ 2.08 & 2.12 & 4.20 & 2.96 & 11.11 & 10.00 & 10.00 \\ 2.89 & 2.96 & 25.77 & 5.90 & 11.11 & 10.00 & 10.00 \\ 4.07 & 4.18 & 4.20 & 1.58 & 11.11 & 10.00 & 10.00 \\ 21.55 & 23.53 & 17.28 & 23.53 & 11.11 & 10.00 & 10.00 \\ 21.55 & 23.53 & 17.28 & 23.53 & 5.56 & 10.00 & 10.00 \\ 7.98 & 8.28 & 4.20 & 16.32 & 11.11 & 10.00 & 10.00 \end{bmatrix} x \begin{bmatrix} 11.83 \\ 27.74 \\ 17.80 \\ 8.04 \\ 27.74 \\ 4.00 \\ 2.85 \end{bmatrix} = \begin{bmatrix} 12.52 \\ 9.71 \\ 8.44 \\ 4.32 \\ 5.59 \\ 9.99 \\ 6.28 \\ 17.81 \\ 16.27 \\ 9.07 \end{bmatrix} \quad (3.9)$$

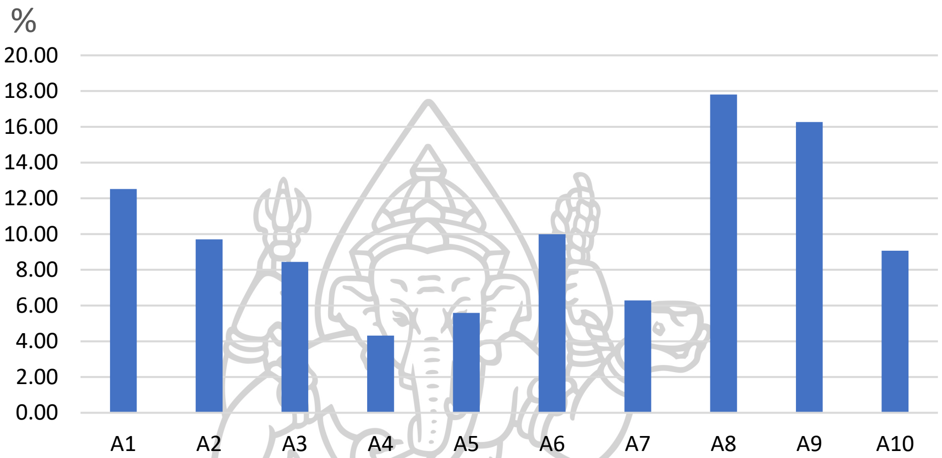

Fig. 15 Weight of power station alternatives

Based on the results from Fig. 15, it can be stated that using the AHP method for a large criteria decision-making problems, alternative 8 is likely to be chosen and is the most beneficial. According to Tables 34-42, as the consistency ratio of AHP is less than 0.10

3.2.2 Applications of classic AHP

Define goals and criteria for decision-making. Group the problem components into levels. The top level is the decision-making goal, level 2 is the criteria, and the last level is the alternative.as shown in Fig. 16.

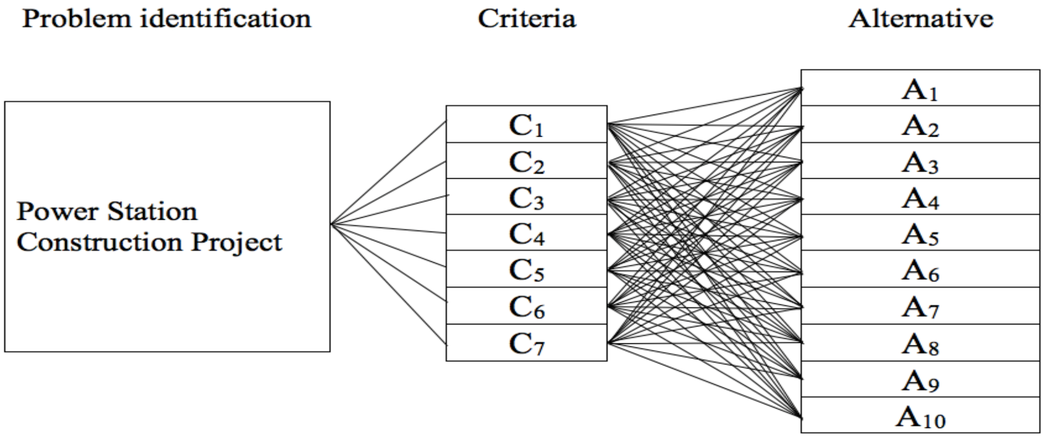

Fig. 16 Structure AHP model for power station construction project

Determine the relative weight of the criteria. The Table 43 presents the values of the comparisons among criteria using the fundamental scale of Thomas L. Saaty [28].

Table 43 Weight of importance and the consistency ratio value

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Weight (%)
C_1	1.00	0.14	0.14	2.00	0.14	3.00	3.00	8.02
C_2	7.00	1.00	1.00	5.00	1.00	5.00	5.00	26.36
C_3	7.00	1.00	1.00	1.00	1.00	5.00	5.00	22.79
C_4	0.50	0.20	1.00	1.00	0.20	1.00	1.00	7.23
C_5	7.00	1.00	1.00	5.00	1.00	5.00	5.00	26.36
C_6	0.33	0.20	0.20	1.00	0.20	1.00	1.00	4.61
C_7	0.33	0.20	0.20	1.00	0.20	1.00	1.00	4.61
CR	0.0883							

From Table 43, the criteria which are most concern are C_2 (Electrical consumption) and C_5 (The forecast of power shortage), the with weight of 26.36 %, followed by C_3 (The problem by the system) with the weight of 22.79 %. C_1

(Electricity income) criterion is ranked thirdly important, with the weight of 8.02 %. The consistency ratio of AHP is 0.0883.

The evaluation of a power station construction project, concerning the seven criteria, taken into consideration, must be unfolded. The assessment for each criterion is shown in Table 44-50.

Table 44 Weight of importance of alternatives and CR for C_1

C_1	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	3.00	3.00	8.00	8.00	5.00	4.00	0.25	0.25	2.00	12.85
A_2	0.33	1.00	2.00	8.00	8.00	5.00	8.00	0.20	0.20	2.00	11.02
A_3	0.33	0.50	1.00	8.00	8.00	6.00	7.00	0.20	0.20	2.00	10.14
A_4	0.13	0.13	0.13	1.00	1.00	0.25	2.00	0.13	0.13	0.20	2.06
A_5	0.13	0.13	0.13	1.00	1.00	0.50	2.00	0.14	0.14	0.20	2.23
A_6	0.20	0.20	0.17	4.00	2.00	1.00	0.50	0.14	0.14	0.20	2.94
A_7	0.25	0.13	0.14	0.50	0.50	2.00	1.00	0.14	0.14	0.25	2.33
A_8	4.00	5.00	5.00	8.00	7.00	7.00	7.00	1.00	1.00	4.00	24.72
A_9	4.00	5.00	5.00	8.00	7.00	7.00	7.00	1.00	1.00	3.00	24.05
A_{10}	0.50	0.50	0.50	5.00	5.00	5.00	4.00	0.25	0.33	1.00	7.67
CR	0.0978										

From Table 44, the most concerned criterion is alternative 8 with the weight of 24.72 %, followed by alternative 9 with the weight of 24.05%. Alternative 1 is ranked thirdly important, with the weight of 12.85%. The consistency ratio of AHP is 0.0978.

Table 45 Weight of importance of alternatives and CR for C_2

C_2	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	3.00	2.00	5.00	4.00	2.00	3.00	0.25	0.25	4.00	11.53
A_2	0.33	1.00	0.50	7.00	5.00	5.00	4.00	0.20	0.25	3.00	9.96
A_3	0.50	2.00	1.00	6.00	3.00	6.00	2.00	0.17	0.17	0.50	7.87
A_4	0.20	0.14	0.17	1.00	0.50	0.50	0.33	0.13	0.13	0.25	1.82
A_5	0.25	0.20	0.33	2.00	1.00	0.50	0.50	0.13	0.13	0.13	2.31
A_6	0.50	0.20	0.17	2.00	2.00	1.00	0.33	0.14	0.14	0.20	2.90
A_7	0.33	0.25	0.50	3.00	2.00	3.00	1.00	0.17	0.17	0.25	4.14
A_8	4.00	5.00	6.00	8.00	8.00	7.00	6.00	1.00	1.00	3.00	25.22
A_9	4.00	4.00	6.00	8.00	8.00	7.00	6.00	1.00	1.00	3.00	24.60
A_{10}	0.25	0.33	2.00	4.00	8.00	5.00	4.00	0.33	0.33	1.00	9.66
CR	0.0894										

From Table 45, the most concerned criterion is alternative 8 with the weight of 25.22%, followed by alternative 9 with the weight of 24.60%. Alternative 1 is ranked thirdly important, with the weight of 11.53%. The consistency ratio of AHP is 0.0894.

Table 46 Weight of importance of alternatives and CR for C_3

C_3	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	1.00	0.25	0.25	1.00	0.50	1.00	2.00	2.00	1.00	8.38
A_2	1.00	1.00	0.50	0.50	1.00	0.33	1.00	0.50	0.50	1.00	5.30
A_3	4.00	2.00	1.00	2.00	2.00	0.25	2.00	0.50	0.50	2.00	10.28
A_4	4.00	2.00	0.50	1.00	2.00	0.25	2.00	0.33	0.33	2.00	8.87
A_5	1.00	1.00	0.50	0.50	1.00	0.17	1.00	0.33	0.33	1.00	4.48
A_6	2.00	3.00	4.00	4.00	6.00	1.00	5.00	4.00	4.00	6.00	27.44
A_7	1.00	1.00	0.50	0.50	1.00	0.20	1.00	0.33	0.33	1.00	4.58

C_3	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_8	0.50	2.00	2.00	3.00	3.00	0.25	3.00	1.00	1.00	7.00	13.29
A_9	0.50	2.00	2.00	3.00	3.00	0.25	3.00	1.00	1.00	7.00	13.29
A_{10}	1.00	1.00	0.50	0.50	1.00	0.17	1.00	0.14	0.14	1.00	4.10
CR	0.0931										

From Table 46, the most concerned criterion is alternative 6 with the weight of 27.44 %, followed by alternative 8 and 9 with the weight of 13.29 %. Alternative 3 is ranked thirdly important, with the weight of 10.28 %. The consistency ratio of AHP is 0.0931.

Table 47 Weight of importance of alternatives and CR for C_4

C_4	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	2.00	3.00	6.00	5.00	3.00	5.00	0.17	0.17	0.25	9.15
A_2	0.50	1.00	3.00	4.00	4.00	2.00	5.00	0.17	0.20	0.33	7.40
A_3	0.33	0.33	1.00	2.00	3.00	2.00	5.00	0.25	0.25	0.14	5.41
A_4	0.17	0.25	0.50	1.00	0.50	0.25	2.00	0.14	0.14	0.17	2.34
A_5	0.20	0.25	0.33	2.00	1.00	0.17	3.00	0.13	0.13	0.14	2.69
A_6	0.33	0.50	0.50	4.00	6.00	1.00	5.00	0.17	0.20	0.25	5.89
A_7	0.20	0.20	0.20	0.50	0.33	0.20	1.00	0.13	0.13	0.17	1.71
A_8	6.00	6.00	4.00	7.00	8.00	6.00	8.00	1.00	1.00	1.00	22.77
A_9	6.00	5.00	4.00	7.00	8.00	5.00	8.00	1.00	1.00	3.00	24.91
A_{10}	4.00	3.00	7.00	6.00	7.00	4.00	6.00	1.00	0.33	1.00	17.73
CR	0.0823										

From Table 47, the most concerned criterion is alternative 9 with the weight of 24.91%, followed by alternative 8 with the weight of 22.77%. Alternative 10 is ranked thirdly important, with the weight of 17.73%. The consistency ratio of AHP is 0.0823.

Table 48 Weight of importance of alternatives and CR for C_5

C_5	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	1.00	0.50	3.00	3.00	3.00	2.00	0.50	3.00	1.00	13.51
A_2	1.00	1.00	1.00	3.00	3.00	1.00	2.00	0.50	4.00	1.00	12.55
A_3	2.00	1.00	1.00	2.00	2.00	1.00	1.00	1.00	3.00	1.00	11.81
A_4	0.33	0.33	0.50	1.00	0.50	0.50	0.33	0.33	1.00	0.33	3.97
A_5	0.33	0.33	0.50	2.00	1.00	0.33	0.50	1.00	4.00	2.00	8.05
A_6	0.33	1.00	1.00	2.00	3.00	1.00	2.00	2.00	2.00	0.50	11.72
A_7	0.50	0.50	1.00	3.00	2.00	0.50	1.00	2.00	6.00	2.00	12.47
A_8	2.00	2.00	1.00	3.00	1.00	0.50	0.50	1.00	2.00	1.00	11.50
A_9	0.33	0.25	0.33	1.00	0.25	0.50	0.17	0.50	1.00	0.25	3.43
A_{10}	1.00	1.00	1.00	3.00	0.50	2.00	0.50	1.00	4.00	1.00	10.99
CR	0.0792										

From Table 48, the most concerned criterion is alternative 1 with the weight of 13.51 %, followed by alternative 2 with weight of 12.55 %. Alternative 7 is ranked thirdly important, with the weight of 12.47 %. The consistency ratio of AHP is 0.0792.

Table 49 Weight of importance of alternatives and CR for C_6

C_6	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	1.00	0.50	2.00	2.00	3.00	2.00	0.33	0.50	1.00	10.02
A_2	1.00	1.00	1.00	2.00	2.00	1.00	3.00	0.33	0.50	1.00	9.41
A_3	2.00	1.00	1.00	4.00	3.00	1.00	2.00	0.50	1.00	1.00	12.30
A_4	0.50	0.50	0.25	1.00	1.00	0.50	1.00	1.00	0.33	0.33	5.47
A_5	0.50	0.50	0.33	1.00	1.00	0.50	1.00	0.33	0.50	0.25	4.68

C_6	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_6	0.33	1.00	1.00	2.00	2.00	1.00	1.00	0.50	1.00	1.00	8.94
A_7	0.50	0.33	0.50	1.00	1.00	1.00	1.00	0.25	0.25	0.50	4.87
A_8	3.00	3.00	2.00	1.00	3.00	2.00	4.00	1.00	1.00	1.00	17.16
A_9	2.00	2.00	1.00	3.00	2.00	1.00	4.00	1.00	1.00	3.00	15.94
A_{10}	1.00	1.00	1.00	3.00	4.00	1.00	2.00	1.00	0.33	1.00	11.19
CR	0.0512										

From Table 49, the most concerned criterion is alternative 8 with weight of 17.16 %, followed by alternative 9 with the weight of 15.94 %. The alternative 3 is ranked thirdly important, with the weight of 12.30 %. The consistency ratio of AHP is 0.0512.

Table 50 Weight of importance of alternatives and CR for C_7

C_7	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (%)
A_1	1.00	1.00	0.50	3.00	3.00	3.00	3.00	0.33	0.33	1.00	9.97
A_2	1.00	1.00	0.33	2.00	2.00	1.00	3.00	0.33	0.33	0.50	7.10
A_3	2.00	3.00	1.00	4.00	4.00	3.00	4.00	1.00	1.00	1.00	16.51
A_4	0.33	0.50	0.25	1.00	1.00	0.50	1.00	0.25	0.25	0.33	3.79
A_5	0.33	0.50	0.25	1.00	1.00	0.33	2.00	0.25	0.25	0.33	4.05
A_6	0.33	1.00	0.33	2.00	3.00	1.00	3.00	0.33	0.33	0.50	6.96
A_7	0.33	0.33	0.25	1.00	0.50	0.33	1.00	0.25	0.25	0.33	3.39
A_8	3.00	3.00	1.00	4.00	4.00	3.00	4.00	1.00	1.00	1.00	17.32
A_9	3.00	3.00	1.00	4.00	4.00	3.00	4.00	1.00	1.00	2.00	18.57
A_{10}	1.00	2.00	1.00	3.00	3.00	2.00	3.00	1.00	0.50	1.00	12.36
CR	0.0233										

From Table 50, the most concerned criterion is alternative 9 with the weight of 18.57%, followed by alternative 8 with the weight of 17.32%. Alternative 3 is ranked thirdly important, with the weight of 16.51%. The consistency ratio of AHP is 0.0233.

Obtaining such results from Tables 44-50, it is now possible to generate matrix A_{ij}^{C}. The columns in matrix C are put into order according to the criteria determined in Table 43, we found $w^T =$ [8.02 26.36 22.79 7.23 26.36 4.61 4.61]. Performing the multiplication of matrix and the vector weight, the preference vector for the ten power station construction project appears according to the following relation:

$$x = A_{ij}^{C} \ x \ w^T = \begin{bmatrix} 12.85 & 11.53 & 8.38 & 9.15 & 13.51 & 10.02 & 9.97 \\ 11.02 & 9.96 & 5.30 & 7.40 & 12.55 & 9.41 & 7.10 \\ 10.14 & 7.87 & 10.28 & 5.41 & 11.81 & 12.30 & 16.51 \\ 2.06 & 1.82 & 8.87 & 2.34 & 3.97 & 5.47 & 3.79 \\ 2.23 & 2.31 & 4.48 & 2.69 & 8.05 & 4.68 & 4.05 \\ 2.94 & 2.90 & 27.44 & 5.89 & 11.72 & 8.94 & 6.96 \\ 2.33 & 4.14 & 4.58 & 1.71 & 12.47 & 4.87 & 3.39 \\ 24.72 & 25.22 & 13.29 & 22.77 & 11.50 & 17.16 & 17.32 \\ 24.05 & 24.60 & 13.29 & 24.91 & 3.43 & 15.94 & 18.57 \\ 7.67 & 9.66 & 4.10 & 17.73 & 10.99 & 11.19 & 12.36 \end{bmatrix} x \begin{bmatrix} 8.02 \\ 26.36 \\ 22.79 \\ 7.23 \\ 26.36 \\ 4.61 \\ 4.61 \end{bmatrix} = \begin{bmatrix} 11.13 \\ 9.32 \\ 10.06 \\ 4.31 \\ 4.53 \\ 11.50 \\ 6.11 \\ 17.93 \\ 15.74 \\ 9.36 \end{bmatrix} \quad (3.10)$$

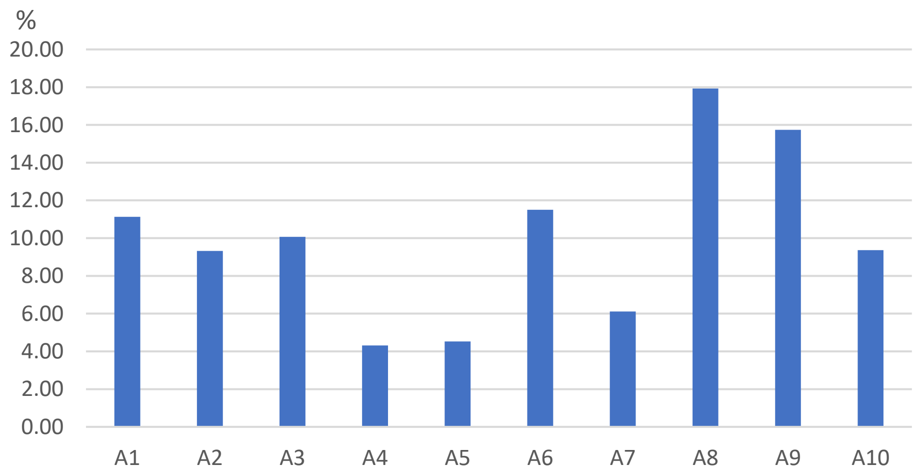

Fig. 17 Weight of power station alternatives for classic AHP

Based on the results from Fig. 17, it can be stated that, using the AHP method for extensive criteria decision-making problems, alternative 8 is likely to be chosen and is the most beneficial. According to Tables 44-50, the consistency ratio of the AHP is less than 0.10.

3.3 Conclusion

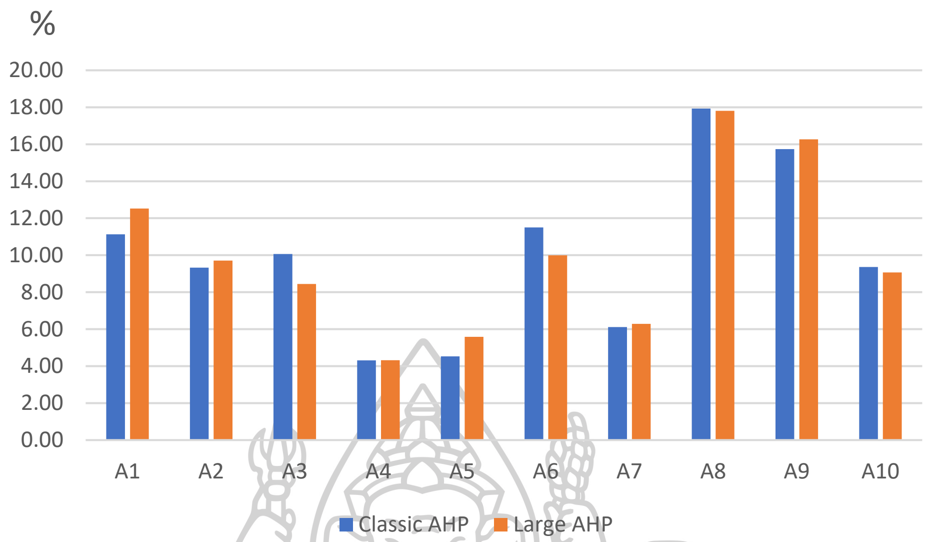

Fig. 18 Comparison of the modified AHP and classic AHP

Fig. 18 shows the weight result of the different alternatives power station construction projects resulting from using classic AHP and modified AHP. It can be seen that all power station construction alternative weights are consistent with each other referred to the Table 34, the weight of importance using classic AHP, and Table 43. The weight of importance calculation using modified AHP is the method of finding the importance of criteria used in considering power station construction project alternatives in which the results from the total display the same order of matter apart from C_6 and C_7; They both have the least weight of significance and result almost at the same values. Regarding Tables 36-39, The weight of importance of each alternative for C_1-C_4 of classic AHP, and Tables 44-47, The weight of importance of each alternative for C_1-C_4 of modified AHP, the orders of importance of both tables go along in the same direction. However, for Tables 40-42, The weight of importance of each alternative for C_5-C_7 of classic AHP, and Tables 48-50, The weight of importance of each alternative for C_5-C_7 of modified AHP, there is a significant difference in the orders of significance due to the reason that, during the modified AHP process, the experts ranked equally on C_5-C_7 criteria for many potential power stations.

The outcome-changing threshold of the classic AHP is slightly larger than the modified AHP. This means that the pairwise comparison approach is more robust. Nevertheless, the modified AHP is more applicable in actual practice. It gives the same as the decision of the classic AHP while demanding less effort. It reduces the action of the decision-makers by 77.08 %, while the classic AHP needs 336 decisions, the modified AHP needs only 77. To express bold evidence, suppose each decision-making needs 1 minute to discuss and make a judgment; 336 decisions expect around 5 hours and a half to conduct. Nonetheless, the modified AHP needs only 1.28 hours to finish with the same result.

3.3.1 Discussion

The present, there are various developed tools to support the decision-making. It is reliable and accurate in multi-criteria decision-making. The AHP applied to the normalize function-based scaling AHP method used in large-scale cases for improves efficiency. The proposed method yielded the same conclusion as the classical AHP, TOPSIS, VIKOR, and ANP while requiring significantly less effort. Furthermore, the threshold of decision changing was not a substantial discrepancy.

3.3.2 Recommendations

In this normalize function-based scaling AHP method, the researcher had to collect data using the workshop method to summarize the scoring. It reduces data variation, missing data

CHAPTER 4

Fuzzy Analytic Hierarchy Processes Analysis

The AHP has been considered one of the highly used multiple-criteria decision-making tools and has been extensively studied in depth until now. The AHP has various applications, like resource allocation of business or public policy, strategic planning, source selection, program selection, and task priority [16]. Jayaraman et al. [14] proposed the MCDM, using a goal programming model, in strategic planning and resource allocation to expand and implement responsible strategies in the long term. The problems in construction management were analyzed, solved, and discussed by adapting and combining MCDM and analytic hierarchy process approaches. From the primary study on related literature, it is found that several studies mention and deploy the combination of fuzzy functions and AHP called the Fuzzy Analytic Hierarchy Process (FAHP). Jayawickrama et al. [17] present a generic model that evaluates the sustainable performance of manufacturing plants using FAHP. This tool can help resolve a variation point or a variability, evaluate, and study the feasibility of the plant operation. Kaganski et al. [18] use the FAHP as a tool to prioritize key performance indicators based on SMARTER criteria and 13 KPIs. The weights for the SMARTER criteria will also be developed. Radziszewska et al. [19] propose that supporting partnership relation management in the implementation of construction projects using FAHP as an adjustment is likely highly advantageous in terms of its duration, cost, quality and safety. Considering the information above, the purpose of this study is to compare the effectiveness of classic

AHP and FAHP in the triangle and trapezoidal models using the weighing results and consistency ratio values on the same data.

4.1 Fuzzy analytic hierarchy process

4.1.1 FAHP for Triangle model

The most straightforward membership functions are formed using straight lines. Of these, the simplest is the triangle membership function. It is nothing more than a collection of three points forming a triangle. The graphical representation of the triangle membership function is shown in Fig. 19. [39]

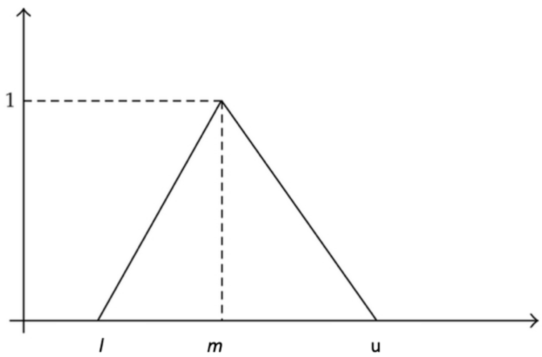

Fig. 19 Triangle fuzzy numbers
source: Chang [50]

The FAHP method presents triangle fuzzy numbers. It can be identified as triple $x = (l, m, u)$, where defines a membership function as [50],

$$\mu(x) = \begin{cases} \dfrac{x}{m-l} - \dfrac{l}{m-l}, & x \epsilon [l, m] \\ \dfrac{x}{m-u} - \dfrac{u}{m-u}, & x \epsilon [m, u] \\ 0, & oterwise \end{cases} \qquad (4.1)$$

A triangle fuzzy number is developed by applying AHP to compare prioritized scales between each criterion, as shown in Table 51.

Table 51 Linguistic terms and the corresponding FAHP for triangle

Saaty scale	Definition	Fuzzy Triangular Scale C_l, C_m, C_u
1	Equal importance (Eq)	(1, 1, 1)
2	Intermediate values (EIW)	(1, 2, 3)
3	Weakly importance (W)	(2, 3, 4)
4	Intermediate values (WIE)	(3, 4, 5)
5	Essentially importance (Es)	(4, 5, 6)
6	Intermediate values (EIV)	(5, 6, 7)
7	Very strongly importance (V)	(6, 7, 8)
8	Intermediate values (VIE)	(7, 8, 9)
9	Extreme importance (Ex)	(9, 9, 9)

Then the judgment, matrix A, which contains comparison value C_{ij} for all $i, j \in \{1, 2, \ldots, \mathrm{n}\}$ is given by (4.2)

$$A = \begin{bmatrix} (C_l, C_m, C_u)_{11} & (C_l, C_m, C_u)_{12} & (C_l, C_m, C_u)_{13} & \cdots & (C_l, C_m, C_u)_{1\mathrm{n}} \\ \frac{1}{(C_l, C_m, C_u)_{21}} & (C_l, C_m, C_u)_{22} & (C_l, C_m, C_u)_{23} & \cdots & (C_l, C_m, C_u)_{2\mathrm{n}} \\ \frac{1}{(C_l, C_m, C_u)_{31}} & \frac{1}{(C_l, C_m, C_u)_{32}} & (C_l, C_m, C_u)_{33} & \cdots & (C_l, C_m, C_u)_{3\mathrm{n}} \\ \vdots & & \vdots & & \vdots \\ \frac{1}{(C_l, C_m, C_u)_{\mathrm{n}1}} & \frac{1}{(C_l, C_m, C_u)_{\mathrm{n}2}} & \frac{1}{(C_l, C_m, C_u)_{\mathrm{n}3}} & \cdots & (C_l, C_m, C_u)_{\mathrm{nn}} \end{bmatrix} \quad (4.2)$$

Normalize each column to get a new judgment, matrix A.

$$A' = \begin{bmatrix} (C_l, C_m, C_u)'_{11} & (C_l, C_m, C_u)'_{12} & \cdots & (C_l, C_m, C_u)'_{1n} \\ (C_l, C_m, C_u)'_{21} & (C_l, C_m, C_u)'_{22} & & (C_l, C_m, C_u)'_{2n} \\ \vdots & & \ddots & \vdots \\ (C_l, C_m, C_u)'_{n1} & (C_l, C_m, C_u)'_{n2} & \cdots & (C_l, C_m, C_u)'_{nn} \end{bmatrix} =$$

$$\begin{bmatrix} (C_l)_{11}/\sum_{i=1}^{n}(C_l)_{i1} \; (C_m)_{11}/\sum_{i=1}^{n}(C_m)_{i1} \; (C_u)_{11}/\sum_{i=1}^{n}(C_u)_{i1} & \cdots & (C_l)_{1n}/\sum_{i=1}^{n}(C_l)_{in} \; (C_m)_{1n}/\sum_{i=1}^{n}(C_m)_{in} \; (C_u)_{1n}/\sum_{i=1}^{n}(C_u)_{in} \\ (C_l)_{21}/\sum_{i=1}^{n}(C_l)_{i1} \; (C_m)_{21}/\sum_{i=1}^{n}(C_m)_{i1} \; (C_u)_{21}/\sum_{i=1}^{n}(C_u)_{i1} & & (C_l)_{2n}/\sum_{i=1}^{n}(C_l)_{in} \; (C_m)_{2n}/\sum_{i=1}^{n}(C_m)_{in} \; (C_u)_{2n}/\sum_{i=1}^{n}(C_u)_{in} \\ \vdots & \ddots & \vdots \\ (C_l)_{n1}/\sum_{i=1}^{n}(C_l)_{i1} \; (C_m)_{n1}/\sum_{i=1}^{n}(C_m)_{i1} \; (C_u)_{n1}/\sum_{i=1}^{n}(C_u)_{i1} & \cdots & (C_l)_{nn}/\sum_{i=1}^{n}(C_l)_{in} \; (C_m)_{nn}/\sum_{i=1}^{n}(C_m)_{in} \; (C_u)_{nn}/\sum_{i=1}^{n}(C_u)_{in} \end{bmatrix} \quad (4.3)$$

where $\sum_{i=1}^{n}(C_l, C_m, C_u)_{ij}$ is the sum of column j of the judgment, matrix A.

Sum up each row of normalized judgment matrix A'to get weight vector V.

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n}\left((C_l)_{1n} / \sum_{i=1}^{n}(C_l)_{in} \quad (C_m)_{1n} / \sum_{i=1}^{n}(C_m)_{in} \quad (C_u)_{1n} / \sum_{i=1}^{n}(C_u)_{in}\right)'_{1j} \\ \sum_{j=1}^{n}\left((C_l)_{2n} / \sum_{i=1}^{n}(C_l)_{in} \quad (C_m)_{2n} / \sum_{i=1}^{n}(C_m)_{in} \quad (C_u)_{2n} / \sum_{i=1}^{n}(C_u)_{in}\right)'_{2j} \\ \vdots \\ \sum_{j=1}^{n}\left((C_l)_{nn} / \sum_{i=1}^{n}(C_l)_{in} \quad (C_m)_{nn} / \sum_{i=1}^{n}(C_m)_{in} \quad (C_u)_{nn} / \sum_{i=1}^{n}(C_u)_{in}\right)'_{nj} \end{bmatrix} \tag{4.4}$$

Define the final normalization weight vector W.

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} v_1 / (3 * \sum_{i=1}^{n} v_i) \\ v_2 / (3 * \sum_{i=1}^{n} v_i) \\ \vdots \\ v_n / (3 * \sum_{i=1}^{n} v_i) \end{bmatrix} \tag{4.5}$$

The overall rating results show that the criteria are ordered with the weights decreasingly. The most important criterion has the largest weight. On the other hand, the least important criteria has the smallest weight.

In the next step, we use the consistency checking method developed by Thomas L. Saaty to determine the consistency ratio. Alonso and Lamata [29] show that it can also be estimated in the following equations.

$$CI = \frac{(\lambda_{max} - n)}{n-1} \tag{4.6}$$

$$RI_n = \frac{(2.7699n - 4.3513 - n)}{n-1} \tag{4.7}$$

$$CR = \frac{CI}{RI_n} < 0.1 \tag{4.8}$$

where CI = Consistency index

λ_{max} = The largest eigenvalue of a matrix

n = Number of elements in the matrix

RI_n = Random index computed for matrices that depend on n.

CR = Consistency ratio

The decision is acceptable if the consistency ratio is less than or equal to 0.10. However, if it is not, the analyst must redo the whole process [30].

4.1.2 FAHP for Trapezoidal model

The trapezoidal membership function is defined by a lower limit l, an upper limit u, a lower support limit m, and an upper support limit n, where $l < m < n < u$. The graphical representation of the trapezoidal membership function is shown in Fig. 20. [40].

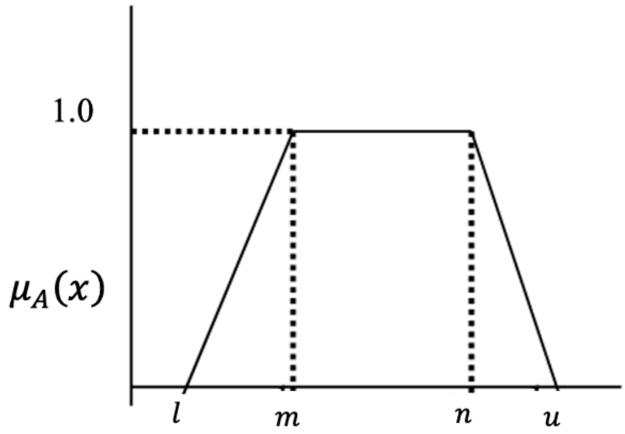

Fig. 20 Trapezoidal fuzzy numbers
source: Tungperachaikul [40]

The FAHP method present a trapezoidal fuzzy numbers. It can be identified as $\mu(x) = (l, m, n, u)$, where defines a membership function as,

$$\mu(x) = \begin{cases} 0, & (x < l)\ or\ (x > u) \\ \dfrac{x-l}{m-l}, & l \leq x \leq m \\ 1, & m \leq x \leq n \\ \dfrac{u-x}{u-n}, & n \leq x \leq u \end{cases} \tag{4.9}$$

A trapezoidal fuzzy number is developed by applying AHP to compare prioritized scales between each criterion, as shown in Table 52.

Table 52 Linguistic terms and the corresponding FAHP for trapezoidal

Saaty scale	Definition	Fuzzy Trapezoidal Scale C_l, C_m, C_n, C_u
1	Equal importance (Eq)	1, 1, 1, 1
2	Intermediate values (EIW)	1, 3/2, 5/2, 3
3	Weakly importance (W)	2, 5/2, 7/2, 4
4	Intermediate values (WIE)	3, 7/2, 9/2, 5
5	Essentially importance (Es)	4, 9/2, 11/2, 6
6	Intermediate values (EIV)	5, 11/2, 13/2, 7
7	Very strongly importance (V)	6, 13/2, 15/2, 8
8	Intermediate values (VIE)	7, 15/2, 17/2, 9
9	Extreme importance (Ex)	8, 17/2, 9, 9

Then the judgment, matrix A, which contains comparison value C_{ij} for all $i, j \in \{1, 2, \ldots, \mathrm{N}\}$ is given by (4.2)

$$A = \begin{bmatrix} (C_l, C_m, C_n, C_u)_{11} & (C_l, C_m, C_n, C_u)_{12} & (C_l, C_m, C_n, C_u)_{13} & \cdots & (C_l, C_m, C_n, C_u)_{1\mathrm{N}} \\ \frac{1}{(C_l, C_m, C_n, C_u)_{21}} & (C_l, C_m, C_n, C_u)_{22} & (C_l, C_m, C_n, C_u)_{23} & \cdots & (C_l, C_m, C_n, C_u)_{2\mathrm{N}} \\ \frac{1}{(C_l, C_m, C_n, C_u)_{31}} & \frac{1}{(C_l, C_m, C_n, C_u)_{32}} & (C_l, C_m, C_n, C_u)_{33} & \cdots & (C_l, C_m, C_n, C_u)_{3\mathrm{N}} \\ \vdots & & \vdots & & \vdots \\ \frac{1}{(C_l, C_m, C_n, C_u)_{\mathrm{N}1}} & \frac{1}{(C_l, C_m, C_n, C_u)_{\mathrm{N}2}} & \frac{1}{(C_l, C_m, C_n, C_u)_{\mathrm{N}3}} & \cdots & (C_l, C_m, C_n, C_u)_{\mathrm{NN}} \end{bmatrix} \quad (4.10)$$

Normalize each column to get a new judgment, matrix A.

$$A' = \begin{bmatrix} (C_l, C_m, C_n, C_u)'_{11} & (C_l, C_m, C_n, C_u)'_{12} & \cdots & (C_l, C_m, C_n, C_u)'_{1N} \\ (C_l, C_m, C_n, C_u)'_{21} & (C_l, C_m, C_n, C_u)'_{22} & & (C_l, C_m, C_n, C_u)'_{2N} \\ \vdots & & \ddots & \vdots \\ (C_l, C_m, C_n, C_u)'_{N1} & (C_l, C_m, C_n, C_u)'_{N2} & \cdots & (C_l, C_m, C_n, C_u)'_{NN} \end{bmatrix} =$$

$$\begin{bmatrix} (C_l)_{11}/\sum_{i=1}^{N}(C_l)_{i1} \; (C_m)_{11}/\sum_{i=1}^{N}(C_m)_{i1} \; (C_n)_{11}/\sum_{i=1}^{N}(C_n)_{i1} \; (C_u)_{11}/\sum_{i=1}^{N}(C_u)_{i1} & \cdots & (C_l)_{1N}/\sum_{i=1}^{N}(C_l)_{iN} \; (C_m)_{1N}/\sum_{i=1}^{N}(C_m)_{iN} \; (C_n)_{1N}/\sum_{i=1}^{N}(C_n)_{iN} \; (C_u)_{1N}/\sum_{i=1}^{N}(C_u)_{iN} \\ (C_l)_{21}/\sum_{i=1}^{N}(C_l)_{i1} \; (C_m)_{21}/\sum_{i=1}^{N}(C_m)_{i1} \; (C_n)_{21}/\sum_{i=1}^{N}(C_n)_{i1} \; (C_u)_{21}/\sum_{i=1}^{N}(C_u)_{i1} & & (C_l)_{2N}/\sum_{i=1}^{N}(C_l)_{iN} \; (C_m)_{2N}/\sum_{i=1}^{N}(C_m)_{iN} \; (C_n)_{2N}/\sum_{i=1}^{N}(C_n)_{iN} \; (C_u)_{2N}/\sum_{i=1}^{N}(C_u)_{iN} \\ \vdots & \ddots & \vdots \\ (C_l)_{N1}/\sum_{i=1}^{N}(C_l)_{i1} \; (C_m)_{N1}/\sum_{i=1}^{N}(C_m)_{i1} \; (C_n)_{N1}/\sum_{i=1}^{N}(C_n)_{i1} \; (C_u)_{N1}/\sum_{i=1}^{N}(C_u)_{i1} & \cdots & (C_l)_{NN}/\sum_{i=1}^{N}(C_l)_{iN} \; (C_m)_{NN}/\sum_{i=1}^{N}(C_m)_{iN} \; (C_n)_{NN}/\sum_{i=1}^{N}(C_n)_{iN} \; (C_u)_{NN}/\sum_{i=1}^{N}(C_u)_{iN} \end{bmatrix} \quad (4.11)$$

Where $\sum_{i=1}^{n}(C_l, C_m, C_n, C_u)_{ij}$ is the sum of column j of the judgment, matrix A.

Sum up each row of normalized judgment matrix A'to get weight vector V.

$$V=\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}=\begin{bmatrix} \sum_{j=1}^{N}((C_l)_{1N}/\sum_{i=1}^{N}(C_l)_{iN} & (C_m)_{1N}/\sum_{i=1}^{N}(C_m)_{iN} & (C_n)_{1N}/\sum_{i=1}^{N}(C_n)_{iN} & (C_u)_{1N}/\sum_{i=1}^{N}(C_u)_{iN})'_{1j} \\ \sum_{j=1}^{N}((C_l)_{2N}/\sum_{i=1}^{N}(C_l)_{iN} & (C_m)_{2N}/\sum_{i=1}^{N}(C_m)_{iN} & (C_n)_{2N}/\sum_{i=1}^{N}(C_n)_{iN} & (C_u)_{2N}/\sum_{i=1}^{N}(C_u)_{iN})'_{2j} \\ & \vdots & & \\ \sum_{j=1}^{N}((C_l)_{NN}/\sum_{i=1}^{N}(C_l)_{iN} & (C_m)_{NN}/\sum_{i=1}^{N}(C_m)_{iN} & (C_n)_{NN}/\sum_{i=1}^{N}(C_n)_{iN} & (C_u)_{NN}/\sum_{i=1}^{N}(C_u)_{iN})'_{Nj} \end{bmatrix} \quad (4.12)$$

Define the final normalization weight vector W.

$$w=\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}=\begin{bmatrix} v_1/(4*\sum_{i=1}^{n} v_i) \\ v_2/(4*\sum_{i=1}^{n} v_i) \\ \vdots \\ v_n/(4*\sum_{i=1}^{n} v_i) \end{bmatrix} \quad (4.13)$$

The overall rating result show that the criteria are ordered with the weights decreasingly. The most important criterium has the largest weight. On the other hand, the least important criterium has the smallest weight.

In the next step, we use the consistency checking method developed by Thomas L. Saaty to determine the consistency ratio. Alonso and Lamata [29] show that it can also be estimated in the following equations.

$$CI=\frac{(\lambda_{max}-N)}{n-1} \quad (4.14)$$

$$RI_n=\frac{(2.7699n-4.3513-N)}{N-1} \quad (4.15)$$

$$CR=\frac{CI}{RI_n}<0.1 \quad (4.16)$$

where CI = Consistency index

λ_{max} = The largest eigenvalue of a matrix

N = Number of elements in the matrix

RI_n = Random index computed for matrices that depend on n.

CR = Consistency ratio

The decision is acceptable if the consistency ratio is less than or equal to 0.10. However, if it is not, the analyst must redo the whole process [30].

4.2 Applications of AHPs

In this study, the researcher uses AHP and FAHP techniques to compare both in Triangle model and in the trapezoidal model by considering the result of the weight of importance and consistency ratio values into account using the same set of information. This is to see which method, AHP or FAHP, is more efficient for comparing medical devices suppliers in line with the need for future use in considering the suppliers from different criteria. There are 5 criteria we used to assess the data and suppliers' efficiency: Price (C_1), Payment terms (C_2), Delivery time (C_3), Service (C_4), Quality (C_5), and 3 medical device suppliers have been chosen to participate in this study.

To define the goal and criteria decision-making, the researcher groups the problem components into levels as follows: level "0" indicates 'the goal' of selecting a new suitable supplier. At level "1", the main criteria are $C_1, C_2, ..., C_n$. Level "2" in the choices of medical device suppliers shown as Supplier 1 (S_1), Supplier 2 (S_2), and Supplier n (S_n). as shown in Fig. 21.

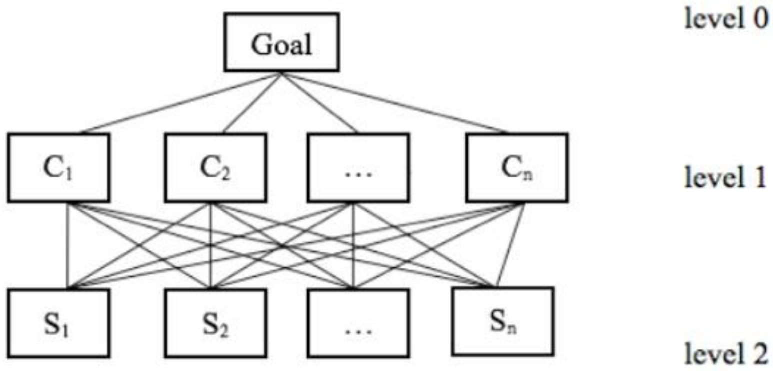

Fig. 21 Structure AHP model for a multi-level hierarchy for supplier selection

4.2.1 Applications of AHP for Classic AHP

To determine the relative weight of the criteria, Table 53 presents the values of the comparisons among criteria, using the fundamental scale of Thomas L. Saaty [28].

Table 53 Weight of importance and the consistency ratio value

Criteria	C_1	C_2	C_3	C_4	C_5	Weight (%)
C_1	1	3	1	1	1/5	14.40
C_2	1/3	1	1/3	3	1/5	10.28
C_3	1	3	1	3	1/3	18.86
C_4	1	1/3	1/3	1	1/5	7.72
C_5	5	5	3	5	1	48.73
CR	0.0986					

From Table 53, the most concerned criterion is C_5 (Quality) with the weight of 48.73 %, followed by C_3 (Delivery time), with the weight of 18.86 %. The C_1 (Price) criterion is ranked thirdly important, with the weight of 14.40 %. The consistency ratio of AHP is 0.0986.

The evaluation of the three medical device suppliers, with concerning the 5 criteria taken into consideration, must be unfolded. The assessment for each criterion is shown in Tables 54-58.

Table 54 Weight of importance of each supplier and CR for C_1

C_1	S_1	S_2	S_3	Weight (%)
S_1	1	7	5	72.35
S_2	1/7	1	1/3	8.33
S_3	1/5	3	1	19.32
CR	0.0559			

From Table 54, the most concerned criterion is supplier 1 with the weight of 72.35 %, followed by supplier 3, with the weight of 19.32 %. Supplier 2 is ranked thirdly important, with the weight of 8.33 %. The consistency ratio of AHP is 0.0559.

Table 55 Weight of importance of each supplier and CR for C_2

C_2	S_1	S_2	S_3	Weight (%)
S_1	1	5	1	45.45
S_2	1/5	1	1/5	9.09
S_3	1	5	1	45.45
CR	0.0000			

From Table 55, the most concerned criteria are suppliers 1 and 3 with the weight of 45.45 %, followed by supplier 2 with the weight of 9.09 %. The consistency ratio of AHP is 0.00.

Table 56 Weight of importance of each supplier and CR for C_3

C_3	S_1	S_2	S_3	Weight (%)
S_1	1	7	3	64.34
S_2	1/7	1	1/5	7.38
S_3	1/3	5	1	28.28
CR	0.0559			

From Table 56, the most concerned criterion is supplier 1 with the weight of 64.34 %, followed by supplier 3 with the weight of 28.28 %. Supplier 2 is ranked thirdly important, with the weight of 7.38 %. The consistency ratio of AHP is 0.0559.

Table 57 Weight of importance of each supplier and CR for C_4

C_4	S_1	S_2	S_3	Weight (%)
S_1	1	1	1/7	11.11
S_2	1	1	1/7	11.11
S_3	7	7	1	77.78
CR	0.0000			

From Table 57, the most concerned criterion is supplier 3 with the weight of 77.78 %, followed by suppliers 1 and 2 with the weight of 11.11%. The consistency ratio of AHP is 0.00.

Table 58 Weight of importance of each supplier and CR for C_5

C_5	S_1	S_2	S_3	Weight (%)
S_1	1	1	1	33.33
S_2	1	1	1	33.33
S_3	1	1	1	33.33
CR	0.0000			

From Table 58, the most concerned criteria are suppliers 1, 2 and 3 with the weight of 33.33 %. The consistency ratio of AHP is 0.00.

Obtaining such results from Tables 54-58, it is now possible to generate matrix A_{ij}^{C}. The columns in matrix C are put in the order of the criteria determined in Table 53; we found $w^T =$ [14.40 10.28 18.86 7.72 48.73]. Performing the multiplication of the matrix and the vector weight, the preference vector for the three supplier structures appears according to the following relation:

$$x = A_{ij}^{C} \text{ x } w^T = \begin{bmatrix} 72.35 & 45.45 & 63.34 & 11.11 & 33.33 \\ 8.33 & 9.09 & 7.38 & 11.11 & 33.33 \\ 19.32 & 45.45 & 28.28 & 77.78 & 33.33 \end{bmatrix} \text{ x } \begin{bmatrix} 14.40 \\ 10.28 \\ 18.86 \\ 7.72 \\ 48.73 \end{bmatrix} = \begin{bmatrix} 44.33 \\ 20.63 \\ 35.04 \end{bmatrix} \quad (4.17)$$

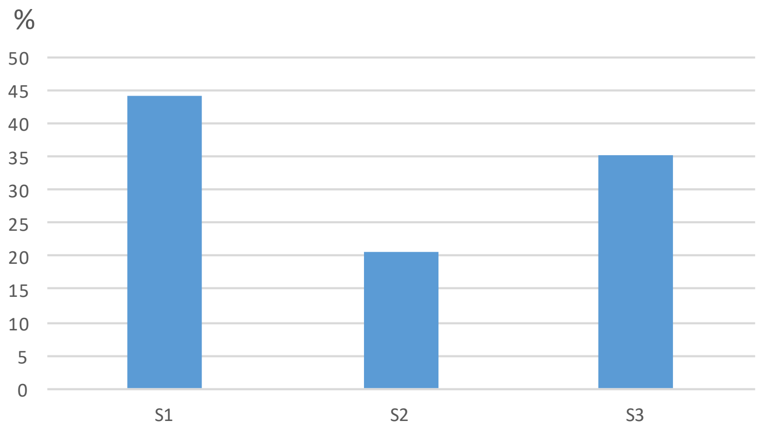

Fig. 22 Resulting weights of suppliers

Based on the results from Fig. 22, it can be stated that using the classic AHP method, supplier I is likely to be chosen and is the most beneficial. According to Tables 53-58, the consistency ratio of AHP is less than 0.01.

4.2.2 Applications of Fuzzy AHP (Triangle)

The pairwise comparison matrix of 5 criteria, in the case of the FAHP (Triangle) is shown in Tables 59–60.

Table 59 The Pairwise comparison for criterion

	C_1	C_2	C_3	C_4	C_5
C_1	Eq	W	Eq	Eq	1/ Es
C_2	1/W	Eq	1/W	W	1/ Es
C_3	Eq	W	Eq	W	1/W
C_4	Eq	1/W	1/W	Eq	1/Es
C_5	Es	Es	W	Es	Eq

Table 59 present the results of the pairwise comparison matrix developed for the present study

Table 60 Weight of importance and the consistency ratio value

	C_1			C_2			C_3			C_4			C_5			Weight
C_1	1	1	1	2	3	4	1	1	1	1	1	1	1/4	1/5	1/6	14.38
C_2	1/2	1/3	1/4	1	1	1	1/2	1/3	1/4	2	3	4	1/4	1/5	1/6	10.49
C_3	1	1	1	2	3	4	1	1	1	2	3	4	1/2	1/3	1/4	18.84
C_4	1	1	1	1/2	1/3	1/4	1/2	1/3	1/4	1	1	1	1/4	1/5	1/6	8.03
C_5	4	5	6	4	5	6	2	3	4	4	5	6	1	1	1	48.26
CR	0.1152															

From Table 60, the most concerned criterion is C_5 (Quality) with the weight of 48.26 %, followed by C_3 (Delivery time), with the weight of 18.84 %. The C_1 (Price) criterion is ranked thirdly important, with the weight of 14.38 %. The consistency ratio of FAHP (Triangle) is more than 0.10.

The evaluation of the three medical device suppliers, concerning the 5 criteria taken into consideration, must be unfolded. The assessment for each criterion is shown in Tables 61-65.

Table 61 Weight of importance of each supplier and CR for C_1

C_1	S_1			S_2			S_3			Weight (%)
S_1	1	1	1	6	7	8	4	5	6	72.08
S_2	1/6	1/7	1/8	1	1	1	1/2	1/3	1/4	8.64
S_3	1/4	1/5	1/6	2	3	4	1	1	1	19.27
CR	0.0856									

From Table 61, the most concerned criterion is supplier 1 with the weight of 72.08 %, followed by supplier 3 with the weight of 19.27%. Supplier 2 is ranked thirdly important, with the weight of 8.64 %. The consistency ratio of FAHP (Triangle) is 0.0856.

Table 62 Weight of importance of each supplier and CR for C_2

C_2	S_1			S_2			S_3			Weight (%)
S_1	1	1	1	4	5	6	1	1	1	45.35
S_2	1/4	1/5	1/6	1	1	1	1/4	1/5	1/6	9.30
S_3	1	1	1	4	5	6	1	1	1	45.35
CR	0.0159									

From Table 62, the most concerned criteria are suppliers 1 and 3 with the weight of 45.35 %, followed by supplier 2 with the weight of 9.30%. The consistency ratio of FAHP (Triangle) is 0.0159.

Table 63 Weight of importance of each supplier and CR for C_3

C_3	S_1			S_2			S_3			Weight (%)
S_1	1	1	1	6	7	8	2	3	4	63.62
S_2	1/6	1/7	1/8	1	1	1	1/4	1/5	1/6	7.54
S_3	1/2	1/3	1/4	4	5	6	1	1	1	28.84
CR	0.0856									

From Table 63, the most concerned criterion is supplier 1 with the weight of 63.62 %, followed by supplier 3 with the weight of 28.84 %. Supplier 2 is ranked thirdly important, with the weight of 7.54 %. The consistency ratio of FAHP (Triangle) is 0.0856.

Table 64 Weight of importance of each supplier and CR for C_4

C_4	S_1			S_2			S_3			Weight (%)
S_1	1	1	1	1	1	1	1/6	1/7	1/8	11.20
S_2	1	1	1	1	1	1	1/6	1/7	1/8	11.20
S_3	6	7	8	6	7	8	1	1	1	77.59
CR	0.0080									

From Table 64, the most concerned criterion is supplier 3 with the weight of 77.59 %, followed by suppliers 1 and 2 with the weight of 11.20 %. The consistency ratio of FAHP (Triangle) is 0.0080.

Table 65 Weight of importance of each supplier and CR for C_5

C_5	S_1			S_2			S_3			Weight (%)
S_1	1	1	1	1	1	1	1	1	1	33.33
S_2	1	1	1	1	1	1	1	1	1	33.33
S_3	1	1	1	1	1	1	1	1	1	33.33
CR	0.0000									

From Table 65, the most concerned criteria are suppliers 1, 2 and 3, with the weight of 33.33 %. The consistency ratio of FAHP (Triangle) is 0.00.

Obtaining such results from Tables 61-65, it is now possible to generate matrix A_{ij}^{C}. The columns in matrix C are put into order according to the criteria determined in Table 60; we found $w^T =$ [14.38 10.49 18.84 8.03 48.26]. Performing the multiplication of the matrix and the vector weight, the preference vector for the three supplier structures appears according to the following relation:

$$x = A_{ij}^{C} \times w^T = \begin{bmatrix} 72.08 & 45.35 & 63.62 & 11.20 & 33.33 \\ 8.64 & 9.30 & 7.54 & 11.20 & 33.33 \\ 19.27 & 45.35 & 28.84 & 77.59 & 33.33 \end{bmatrix} \times \begin{bmatrix} 14.38 \\ 10.49 \\ 18.84 \\ 8.03 \\ 48.26 \end{bmatrix} = \begin{bmatrix} 44.10 \\ 20.63 \\ 35.27 \end{bmatrix} \quad (4.18)$$

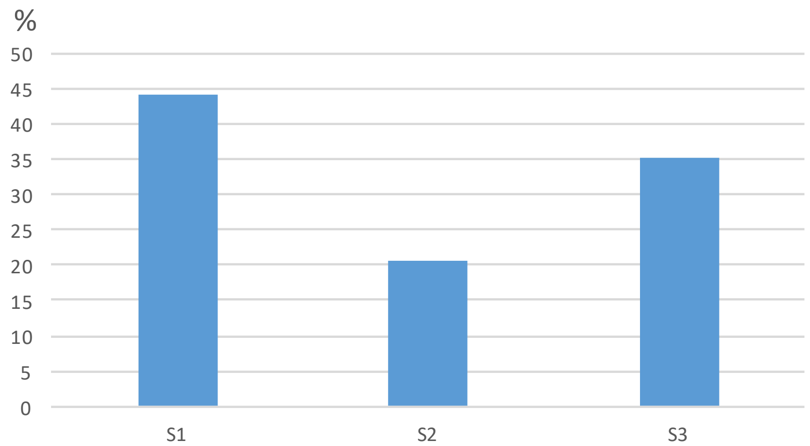

Fig. 23 Resulting weight of each supplier

Based on the results from Fig. 23, it can be stated that using FAHP (Triangle) method, supplier I is likely to be chosen and is the most beneficial. In Table 60, the consistency ratio of FAHP (Triangle) is more than 0.10.

4.2.3 Applications of Fuzzy AHP (Trapezoidal)

The pairwise comparison matrix of 5 criteria in the case of the FAHP (Trapezoidal) is shown in Tables 66–67.

Table 66 Pairwise comparison for criterion

	C_1	C_2	C_3	C_4	C_5
C_1	Eq	W	Eq	Eq	1/ Es
C_2	1/W	Eq	1/W	W	1/ Es
C_3	Eq	W	Eq	W	1/W
C_4	Eq	1/W	1/W	Eq	1/Es
C_5	Es	Es	W	Es	Eq

Table 66 present the results of the pairwise comparison matrix developed for the present study.

Table 67 Weight of importance and the consistency ratio value

	C_1				C_2				C_3				C_4				C_5				W (%)
C_1	1	1	1	1	2	5/2	7/2	4	1	1	1	1	1	1	1	1	1/4	2/9	2/11	1/6	14.38
C_2	1/2	2/5	2/7	1/4	1	1	1	1	1/2	2/5	2/7	1/4	2	5/2	7/2	4	1/4	2/9	2/11	1/6	10.47
C_3	1	1	1	1	2	5/2	7/2	4	1	1	1	1	2	5/2	7/2	4	1/2	2/5	2/7	1/4	18.84
C_4	1	1	1	1	1/2	2/5	2/7	1/4	1/2	2/5	2/7	1/4	1	1	1	1	1/4	2/9	2/11	1/6	8.00
C_5	4	9/2	11/2	6	4	9/2	11/2	6	2	5/2	7/2	4	4	9/2	11/2	6	1	1	1	1	48.29
CR	0.1140																				

From Table 67, the most concerned criterion is C_5 (Quality) with the weight of 48.29 %, followed by C_3 (Delivery time) with the weight of 18.84 %. The C_1 (Price) criterion is ranked thirdly important, with the weight of 14.38 %. The consistency ratio of FAHP (Trapezoidal) is more than 0.01

The assessment of the three medical device suppliers, concerning the 5 criteria taken into consideration, must be unfolded. The evaluation for each criterion is shown in Tables 68-72.

Table 68 Weight of importance of each supplier and CR for C_1

C_1	S_1				S_2				S_3				W (%)
S_1	1	1	1	1	6	13/2	15/2	8	4	9/2	11/2	6	72.10
S_2	1/6	2/13	2/15	1/8	1	1	1	1	1/2	2/5	2/7	1/4	8.62
S_3	1/4	2/9	2/11	1/6	2	5/2	7/2	4	1	1	1	1	19.28
CR	0.0834												

From Table 68, the most concerned criterion is supplier 1 with the weight of 72.10 %, followed by supplier 3 with the weight of 19.28 %. Supplier 2 is ranked thirdly important, with the weight of 8.62 %. The consistency ratio of FAHP (Trapezoidal) is 0.0834.

Table 69 Weight of importance of each supplier and CR for C_2

C_2	S_1				S_2				S_3				W (%)
S_1	1	1	1	1	4	9/2	11/2	6	1	1	1	1	45.36
S_2	1/4	2/9	2/11	1/6	1	1	1	1	1/4	2/9	2/11	1/6	9.28
S_3	1	1	1	1	4	9/2	11/2	6	1	1	1	1	45.36
CR	0.0148												

From Table 69, the most concerned criteria are suppliers 1 and 3 with the weight of 45.36 %, followed by supplier 2 with weight of 9.28 %. The consistency ratio of FAHP (Trapezoidal) is 0.0148.

Table 70 Weight of importance of each supplier and CR for C_3

C_3	S_1				S_2				S_3				W (%)
S_1	1	1	1	1	6	13/2	15/2	8	2	5/2	7/2	4	63.67
S_2	1/6	2/13	2/15	1/8	1	1	1	1	1/4	2/9	2/11	1/6	7.53
S_3	1/2	2/5	2/7	1/4	4	9/2	11/2	6	1	1	1	1	28.80
CR	0.0834												

From Table 70, the most concerned criterion is supplier 1 with the weight of 63.67 %, followed by supplier 3 with the weight of 28.80 %. Supplier 2 is ranked thirdly important, with the weight of 7.53 %. The consistency ratio of FAHP (Trapezoidal) is 0.0834.

Table 71 Weight of importance of each supplier and CR for C_4

C_4	S_1				S_2				S_3				W (%)
S_1	1	1	1	1	1	1	1	1	1/6	2/13	2/15	1/8	11.20
S_2	1	1	1	1	1	1	1	1	1/6	2/13	2/15	1/8	11.20
S_3	6	13/2	15/2	8	6	13/2	15/2	8	1	1	1	1	77.60
CR	0.0074												

From Table 71, the most concerned criterion is supplier 3 with the weight of 77.60 %, followed by suppliers 1 and 2 with the weight of 11.20 %. The consistency ratio of FAHP (Trapezoidal) is 0.0074.

Table 72 Weight of importance of each supplier and CR for C_5

C_5	S_1				S_2				S_3				W (%)
S_1	1	1	1	1	1	1	1	1	1	1	1	1	33.33
S_2	1	1	1	1	1	1	1	1	1	1	1	1	33.33
S_3	1	1	1	1	1	1	1	1	1	1	1	1	33.33
CR	0.0000												

From Table 72, the most concerned criteria are suppliers 1, 2 and 3 with the weight of 33.33 %. The consistency ratio of FAHP (Trapezoidal) is 0.00.

Obtaining such results from Tables 68-72, it is now possible to generate matrix A_{ij}^{C}. The columns in matrix C are put into order according to the criteria determined in Table 67; we found w^T = [14.38 10.47 18.84 8.00 48.29]. Performing the multiplication of the matrix and the vector weight, the preference vector for the three supplier structures appears according to the following relation:

$$x = A_{ij}^{C} \text{ x } w^T = \begin{bmatrix} 72.10 & 45.36 & 63.67 & 11.20 & 33.33 \\ 8.62 & 9.28 & 7.53 & 11.20 & 33.33 \\ 19.28 & 45.36 & 28.80 & 77.60 & 33.33 \end{bmatrix} x \begin{bmatrix} 14.38 \\ 10.47 \\ 18.84 \\ 8.00 \\ 48.29 \end{bmatrix} = \begin{bmatrix} 44.11 \\ 20.63 \\ 35.26 \end{bmatrix} \quad (4.19)$$

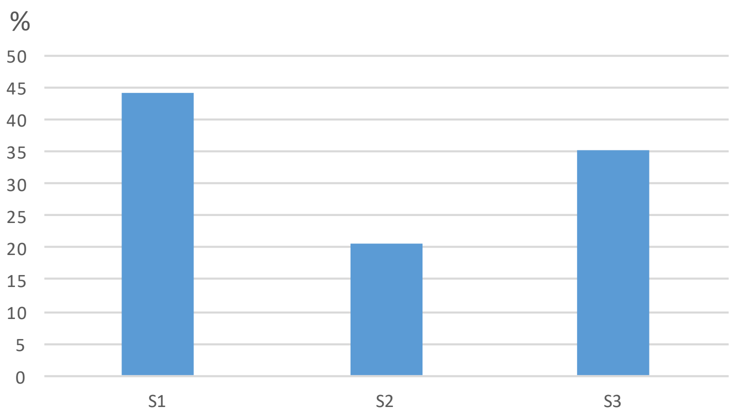

Fig. 24 Resulting weight of each supplier

Based on the results from Figure 24, it can be stated that using FAHP (Trapezoidal) method, supplier I is likely to be chosen and is the most beneficial. In Table 67, the consistency ratio of FAHP (Trapezoidal) is more than 0.10.

4.3 Conclusion

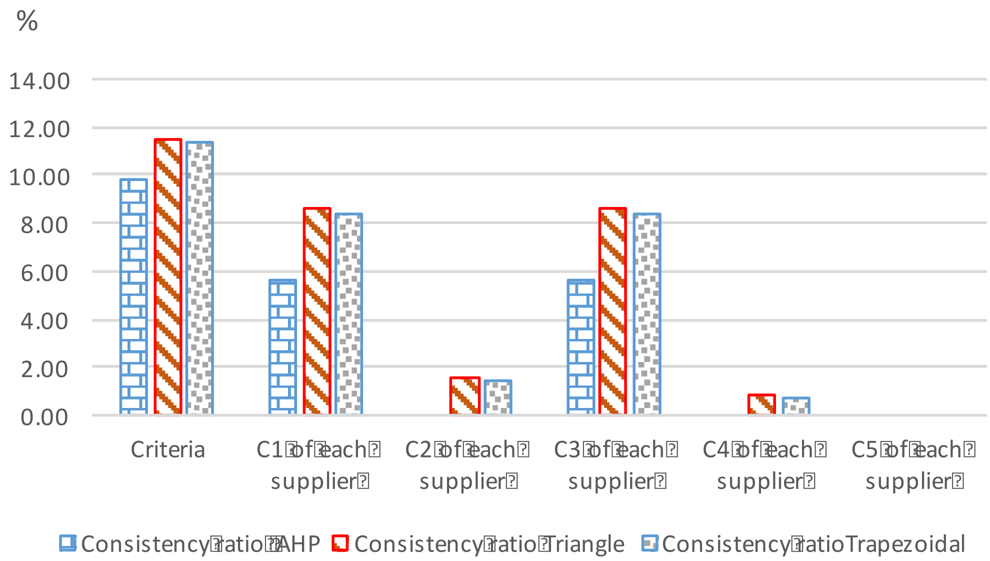

Fig. 25 Consistency ratio comparison

Based on Figure 25, the study has found that consistency ratio values obtained from comparing of classic AHP and FAHP (Triangle) using 5 criteria are different. The consistency ratio of classic AHP is 0.09, which is less than expected 0.10. Meanwhile, the consistency ratios of FAHP are 0.1152 (Triangle) and 0.1140 (Trapezoidal), which is more than an acceptable 0.10. From the comparison between classic AHP and FAHP in parallel based on the 5 criteria: price (C_1), payment terms (C_2), delivery time (C_3), service (C_4) and quality (C_5), it appears that consistency ratio values of both classic AHP and FAHP (Triangle, Trapezoidal) are relatively close. However, there is a difference found as classic AHP's consistency ratio value is less than that FAHPs (Triangle, Trapezoidal) in this study studying the selection of the best medical device supplier candidate.

This study aims to compare 2 decision-making methodologies, classic AHP and FAHP (Triangle, Trapezoidal), used in choosing the preferable medical devices. Price (C_1), payment terms (C_2), delivery time (C_3), service (C_4) and quality (C_5) of 3 medical device suppliers are the main criteria used in the study. The finding shows that both methodologies present similar weights and results in assessing 3 selected medical device supplier candidates. However, the difference can be found in the consistency ratio values of both AHP and FAHP. The consistency ratio value of classic AHP is lower than that of FAHP. Therefore, FAHP (Triangle, Trapezoidal) is more effective as it can rapidly detect and analyze the consistency ratio of classic AHP. Moreover, FAHP is less biased in the parallel comparison using the 5 criteria in FAHP (Triangle, Trapezoidal) method, and a new calculation is recommended.

In conclusion, in case one needs the calculation with less bias, a user should consider FAHP (Triangle) method, as FAHP (Triangle) allow the user to detect and analyze consistency ratio more rapidly. Still, one must accept that it involves more complicated calculation which is considerably recommended for the amateur assessor with authority to approve such vendor. At the same time, classic AHP is suitable for assessors with excessive experience. The researcher applied the FAHP method to 2 study, Solving Supplier Selection for the Photovoltaic System using Fuzzy Analytic Hierarchy Process, and An Application of Fuzzy-AHP Approach to a Product Variety Management Problem. Which, the consistency ratio values of triangle is the most sensitive.

CHAPTER 5

Conclusion

Every day, people make decisions, either on significant or minor issues. However, it is required that the decisions made must be logical enough to generate and ensure good outcomes, as well as to identify alternatives and to come up with information that is beneficial for the organization or the community as a whole and is consistent with the rules and decisions that are well-timed. During the decision-making process, the decision maker should not only be concerned on the benefits and overlook the cause of the basis. Direct and indirect effects after the decision have been made can lead to failing judgment due to the inadequate amount of information received and the pressure the decision-makers have to undergo. The authority should foresee the opportunities and possibilities using their own experiences. When we make a decision, our decision is based mainly on our instinct and ordinary senses. Complex judgment should be made under systematic and logical thinking procedure and other appropriate supporting methods because, to make a difficult decision, there are essential criteria that need to be considered, such as the technically called criteria, concepts, and methodologies of multiple-criteria decision-making. The MCDM method is the consolidation of alternative assessment and the comparison of possible alternatives in different criteria. The comparisons of each option are measured by assessing their appeal according to each criterion, prioritizing the reliability in ranking to determine the weight of each criterion.

In this research, the writer started with literature reviews of AHP. The AHP is one of the MCDM's tools developed by

Thomas L. Saaty in 1970 and the most well-known multi-criteria decision analysis. It was created as a measurement through pairwise comparisons and depends on the decisions of experts to derive priority scales and serves the purpose of classifying the problem into more minor criteria and, later, evaluating the elements hierarchically using mathematics and psychology principles that are related to the ranking of crucial factors during the decision-making and the comparison of a pair of clusters. The AHP is used to hierarchically weigh each element in number according to each element's ranking.

In the first part of this study, the researcher requires the development of a new comparison procedure of an analytic hieratical process to make it convenient to use the AHP analysis to apply cases with large criteria. The proposed AHP and the scoring methods will be improved to make it easier for experts. The technique is called "Normalize function-based scaling AHP". The procedure entails a hierarchical breakdown of the main evaluation problem into more manageable and evaluable subproblems. Given that AHP considers the expressed preferences at each phase, there is no need to estimate a utility function explicitly. The drawback of AHP is that it needs a considerable number of pair-wise comparisons even on a medium-sized problem, says 7 alternatives and 5 criteria. However, in real-world problems, we may face up to 20 alternatives with 10 criteria. It is utmost impossible to employ the AHP. The researchers proposed a novel technique by borrowing the idea of the Likert scale but employing a 1 to 9 scale. The modified techniques are based on the concept of relative criteria scoring and the matrix of comparative criteria scoring. To express our approach's performance, the large-scale multi-criteria decision is used to analyze of the power station construction project selection. There were 10 alternatives and 7 criteria to get through the course of the decision. By comparing the proposed method with the classic AHP with a clustering technique, the proposed method yielded the same conclusion as the classic AHP while requiring significantly less effort. Furthermore, the threshold of decision changing was not a substantial discrepancy.

In the second part of this study, the researcher shows that there are many fuzzy functions; for example, the triangular function and the trapezoidal function. The problem is which function is suitable for a specific AHP based on the decision problem. In other words, the problem by FAHP and function will be most exact for the problem. Therefore, this research wants to increase the performance of FAHP methods. This study compares 2 decision-making methodologies, classic AHP and FAHP (Triangle, Trapezoidal), in the case of choosing the preferable medical devices using the weighing results and consistency ratio values on the same data in the case of medical devices suppliers. This is for us to consider which methodology is the most effective, meet-the-need, and adaptable given multiple-criteria decision-making. The result, in case, one needs the calculation with less bias, a user should consider FAHP (Triangle) method, as FAHP (Triangle) allow the user to detect and analyze consistency ratio more rapidly. Still, one must accept that it involves more complicated calculation which is considerably recommended for the amateur assessor with authority to approve such vendor. At the same time, classic AHP is suitable for assessors with excessive experience.

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