EFFECTS OF SAFE HAVEN STRATEGY AND HERD BEHAVIOR ON FINANCIAL BUBBLE.

By

MR. Sorathan JUANJENKIT

A Thesis Submitted in Partial Fulfillment of the Requirements

for Master of Science MATHEMATICS

Department of MATHEMATICS

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Effects of safe haven strategy and herd behavior on financial bubble.

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Abstract

Safe-haven strategy is usually used to reduce the risk among the market turbulence. It is hypothesized that inclusion of safe-haven asset may reduce the volatility during the bubble. In this study, we propose the new model of financial bubble that generalizes the previous models by adding the safe-haven asset that interacts with the behavioral change of investors. The stability condition is derived to confine the parameter space avoiding the stable fixed point. The numerical results are used to calculate the amplitude and duration of bubbles. The effect of involved parameters are analyzed. This result indicates that information from a safe-haven asset model based on mean reversion helps reduce the severity of financial bubbles resulting from herd behavior of profit seekers in the market. Additionally, it suggests that if these profit seekers consistently use data from safe-haven assets in the market, the severity of financial bubbles would decrease significantly compared to when profit seekers are interested in safe-haven assets only during crisis events.

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Chapter 1

Introduction

Financial bubbles are economic phenomena that have occurred multiple times in history. The definition or description of financial bubbles and the process of their bursting continue to vary and have diverse interpretations. For instance, a definition related to financial bubbles by Didier Sornette suggests that if the price of an asset experiences rapid growth beyond exponential, there is a possibility that the asset may become a financial bubble [10]. Another definition highlights that financial bubbles and the bursting of financial bubbles are temporary events where asset prices deviate and fluctuate around their fundamental value temporarily [8]. One prominent example of a financial bubble event is the Subprime Crisis of 2008. According to 'Review of economic bubble (2016)' [5], the crisis was initiated by a continuous increase in real estate accompanied by loose monetary policies of central banks and governments, which reduced interest rates to encourage more people to own real estate. Additionally, the softening of lending standards brought subprime borrowers into the market. All these factors compounded the growth of real estate, leading people to speculate and invest more, resulting in skyrocketing real estate prices. While everyone was enjoying the prosperity of life, some events were unfolding in the background. 'Inflation' has started creeping in gradually. The low-interest rates, combined with subprime borrowers, led to people defaulting on their loans, and debts began to pile up rapidly. Many homes were foreclosed by banks and released into the market simultaneously with decreased consumer spending. People panicked and wanted to minimize their losses as much as possible, but it was too late.

The research about the financial bubble has been conducted and explored from various perspectives in recent years [12], [7], [10] and [4]. Questions such as where financial bubbles originate, how the mechanics of financial bubbles work, when financial bubbles form and burst, or what factors are related to the occurrence or size of financial bubbles are central to current research. These questions were addressed through various disciplines. For instance, [2] suggested that risky monetary policies by governments and central banks are factors in the emergence of profit-seeking bubbles in the market. Thomas Lux states that financial bubbles arise from the collective behavior and sequential actions of investors in buying or selling until an imbalance occurs between buying and selling demand [8]. What supports the readiness in the behavior of investors to follow each other is fundamental economic variables such as actual returns. While the previous study highlighted the possible influence of herding behavior and the feedback of price during the bubble event, hedging strategy that helps to minimize and offset risks within the portfolio of investors was neglected. Financial hedging is more common amongst short-term noised traders, as market volatility tends to increase. However, research analyzing the impact of other assets on the financial bubble of another asset is relatively limited. Which asset is most important to people in the market?

According to Baur and Lucey (2010), safe-haven is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio in times of market stress or turmoil. As an example, gold or land, which are well-known and have long lasting value over time may be considered as safe-haven assets for stock trading and other risky asset. A safe-haven asset must therefore be some asset that holds its value in 'stormy weather' or adverse market conditions" [3]. For some profit-seeking investors, using safe haven assets to hedge against risk is one of the investment and risk mitigation strategies [1]. In some cases, it is not necessary to include the safe-haven asset into their portfolio but use safe-haven asset data, price volatility of assets, or market returns to make trading decisions in other profit-generating assets. If this is a case during the financial bubble event, how the bubble pattern changes or conditioned might be a crucial issue. In this study, we aim to address these questions and will provide insight into what happens when profit-seeking investors in the market employ strategies and track the price movements of profit-generating assets. How will the financial bubble of those assets develop, shrink, or expand, and what impact will it have on the severity of the economic aftermath?

Due to the wide variety of valuable researches on financial bubbles, all accompanied by various definitions of bubbles, we must choose just one that we deem suitable as a solid foundation to begin addressing the questions. This study extends Thomas Lux's model that emphasized on the impact of Herding behavior on Bubble, and Crash. Herding behavior was explained as events stemming from the collective behavior and sequential actions of investors in buying or selling, leading to an imbalance between buying and selling demand. What reinforces the readiness in investors' behavior to follow each other is fundamental economic variables such as actual returns. This type of model will be integrated with the model of safe-haven asset that we will present in the next section. The model results will be subsequently used to investigate and compare the effect of crucial parameters.

Chapter 2

Models

2.1 Models

2.1.1 Review of Lux's Model

First, we will present for mutual understanding the characteristics of the market under consideration and the definition of the financial bubble based on the previous study. A key feature of the market is that profit-seekers in the market exhibit a behavior known as herd behavior, wherein profit-seekers tend to follow the direction of the crowd in one direction. We presume a market population consisting of a total of $2N$ market participants. Within this population, individuals are divided into two ideological groups: those who view the market negatively, denoted as n_-, representing individuals predisposed to selling assets, and those who view the market positively, denoted as n_+, representing individuals inclined to purchase assets. Additionally, investors are assumed to make buy or sell decisions based on the contagion process, where each individual is immediately prepared to switch from their current group to the larger or predominant group. We introduce the superiority of each group's population with $x = (n_+ - n_-)/2N$, where x is within the range $[-1, 1]$. In cases where $x > 0$, it indicates a prevailing demand for buying assets in the market; conversely, $x < 0$ denotes a predominance of selling. When $x = 0$, it signifies market equilibrium, while $x = 1$ and $x = -1$ represent extreme cases where all market participants converge on the same perspective.

Next, our focus shifts to the properties of market participants, specifically herd behavior or the contagion process within the market. In the market under consideration, we assume that individuals' decisions depend on others within the market, meaning each market participant's decision to buy or sell assets depends on the prevailing sentiment or noise in the market. We further assume that at any given moment, individuals in the market have a probability of switching from being buyers to sellers or vice versa, denoted as p_{-+} and p_{+-}, respectively. Conversely, in the opposite direction, we have p_{+-} and p_{-+}, which, combined with the contagion process, are determined by the collective sentiment of market participants x. Thus, we define $p_{-+} = p_{-+}(x)$ and $p_{+-} = p_{+-}(x)$ based on the overall market sentiment x.

Since there are the probabilities of the transition between optimistic one and pessimistic one, such that we are starting to consider the change of average disposition x. Consequently, we expect fraction $n_{-}\mathrm{p}_{+-}$ to switch from the n_{-} to the n_{+} group which means those who are pessimistic traders turn to an optimistic attitude with probability p_{+-}, and vice versa. From this it follows that the change in time of the number of optimistic and pessimistic traders is : $dn_{+}/dt = n_{-}\mathrm{p}_{+-} - n_{+}\mathrm{p}_{-+}$ and $dn_{-}/dt = n_{+}\mathrm{p}_{-+} - n_{-}\mathrm{p}_{+-}$. Including with n and x that we defined:

$$\begin{aligned} dx/dt &= [(N-n)\mathrm{p}_{+-}(x) - (N+n)\mathrm{p}_{-+}(x)]/N \\ &= (1-x)\mathrm{p}_{+-}(x) - (1+x)\mathrm{p}_{-+}(x). \end{aligned} \tag{2.1}$$

We note that the original arguments serve the stochastic model. However, the derivation of this equation was carried out via the Master equation. To grasp the very idea of how the original basic stochastic process was approximated as an approximation to the change in time of the mean value of the opinion index x which is equation (2.1), it will be explained in the following operation.

First, we can intentionally select a time scale such that only 'nearest neighbor transitions' occur, meaning the likelihood of simultaneous movements by multiple members of the population at any given moment is minimal. We define $n = 0.5(n_+ - n_-)$. We then define the probability that the distribution of the population of speculators changes from $\{n_+, n_-\}$ to $\{n_+ - 1, n_- + 1\}$, or equivalently, from $\{n\}$ to $\{n - 1\}$ as:

$$w_{-+}(n/N) = n_+ p_{-+}(n/N) \tag{2.2}$$

Vice versa, the probability from $\{n\}$ to $\{n + 1\}$ as:

$$w_{+-}(n/N) = n_- p_{+-}(n/N) \tag{2.3}$$

Additionally, if the population size is sufficiently large to permit a continuous-time approximation, the following Master equation can be written to describe the change in the probability distribution $\mathbf{P}(n; t)$ over time:

$$\begin{aligned} d\mathbf{P}(n;t)/dt = [&w_{+-}(n-1)\mathbf{P}(n-1;t) + w_{-+}(n+1)\mathbf{P}(n+1;t)] \\ &- [w_{+-}(n)\mathbf{P}(n;t) + w_{-+}(n)\mathbf{P}(n;t)]. \end{aligned} \tag{2.4}$$

The mean value of $\{n\}$ is defined by:

$$\bar{n}_t = \sum_{n=-N}^{N} n\mathbf{P}(n;t). \tag{2.5}$$

Its change over time is given by:

$$\begin{aligned} d\bar{n}_t/dt &= \sum_{n=-N}^{N} n\mathbf{P}(n;t)/dt \\ &= \sum_{n=-N}^{N} [w_{+-}(n) - w_{-+}(n)]\mathbf{P}(n;t) = \overline{w_{+-}(n) - w_{-+}(n)}. \end{aligned} \tag{2.6}$$

An approximation of the right-hand side (RHS) of equation (2.6) can be made by considering the first term in the Taylor series expansion around $\bar{n}_t$, resulting in the closed expression:

$$d\bar{n}_t = w_{+-}(\bar{n}) - w_{-+}(\bar{n}). \tag{2.7}$$

Dividing by N and substituting the definitions (2.2) and (2.3), equation (2.7) is translated into a dynamic equation for the mean value of the opinion index x, which is:

$$\begin{aligned} d\bar{x}/dt &= [(N-\bar{n})\mathrm{p}_{+-}(\bar{n}/N) - (N+\bar{n})\mathrm{p}_{-+}(\bar{n}/N)]/N \\ &= (1-\bar{x})\mathrm{p}_{+-}(\bar{x}) - (1+\bar{x})\mathrm{p}_{-+}(\bar{x}) \end{aligned} \tag{2.8}$$

Suppressing the bars, (2.8) is the same as (2.1) in the main text. This operation has converted the stochastic dynamics into a quasi-deterministic one, significantly simplifying analysis.

The transition probabilities will be specified in order to perceive how (2.1) potentially-describes. Note that the requirements for p_{+-} and p_{-+} is, (1) all transition probabilities have to be positive, (2) if the prevailing disposition of the population is already optimistic then $\mathrm{p}_{-+} > \mathrm{p}_{+-}$. Moreover, it seems reasonable to assume that $\mathrm{dp}_{+-}/\mathrm{p}_{+-} = a\ dx$, that is the relative changes in probability to switch from pessimism to optimism increases linearly with-changes in x, and vice versa $\mathrm{dp}_{-+}/\mathrm{p}_{-+} = -a\ dx$, where a is a constant. These assumptions may suggest the following functional form commonly chosen in the related literature:

$$\mathrm{p}_{+-}(x) = ve^{ax}, \ \ \mathrm{p}_{-+}(x) = ve^{-ax}. \tag{2.9}$$

Here, a gives a measure for the strength of herd behavior and $a > 0$, v is a variable for the speed of change and $v \in [0, 0.5]$ ($x = 0$, balanced disposition we have $\mathrm{p}_{+-} = \mathrm{p}_{-+} = v > 0$). This means that a little change from equilibrium point is the starting point of herd behavior.

Follow by properties of the hyperbolic sine and cosine and this specification of transition rates the time development of the mean value of the index x becomes:

$$\begin{aligned} dx/dt &= (1-x)ve^{ax} - (1+x)ve^{-ax} = 2v[\sinh(ax) - x\cosh(ax)] \\ &= 2v[\tanh(ax) - x]\cosh(ax). \end{aligned} \tag{2.10}$$

The equation (2.10) represents changes in the majority Sentiment of the market. As the price of focusing securities changes according to the excess demand, the further assumption relies on the direct proportionality of the excess demand on the market sentiment and the deviation of price from the fundamental value. These two factors used the different proportionality constants that distinguishes between the trading volume of speculative investors and of fundamentalists. The corresponding dynamics are given by

$$\begin{aligned} \frac{dx}{dt} &= 2v[\tanh(a_1\dot{p}/v + a_2x) - x]\cosh(a_1\dot{p}/v + a_2x), \\ \frac{dp}{dt} &= \beta[xT_N + T_F(p_f - p)], \end{aligned} \tag{2.11}$$

where dp/dt and $\dot{p}$, representing the rate of change in the price of the underlying asset. According to dp/dt, price changes are driven by the excess demand of two groups of speculators: Fundamental traders, who trade based on the perceived discrepancy between current prices and fundamental values, and Noise traders, who follow others' actions. The excess demand of Fundamental traders is denoted by $T_F(p_f - p)$, where T_F is the trading volume of Fundamental traders, and p_f is the Fundamental price of the underlying asset. On the other hand, Noise traders' excess demand is represented by xT_N, with T_N being the trading volume of Noise traders. a_1 is weight factor describing how much information investors try to draw from price and a_2 is weight factor describing how much information investors drawn from the behavior of others.

Furthermore, the contagion process and price dynamics have different mean time lags, denoted by $1/v$ and $1/\beta$, respectively. Assuming instantaneous market clearing, the equation implies that $p = p_f + (T_N/T_F)x$ and $\dot{p} = (T_N/T_F)\dot{x}$, where the expected returns influence the readiness of profit-seekers to follow suit in the market, and $\dot{x}$ represents the rate of change in x. This readiness is influenced by the cumulative difference between the true returns of the underlying asset and the expected returns in the market.

$$\begin{aligned}\frac{dx}{dt} &= 2v[\tanh(a_0 + a_2 x) - x]\cosh(a_0 + a_2 x),\\ \frac{da_0}{dt} &= \tau\left[\frac{r + \tau^{-1}(T_N/T_F)\dot{x}}{p_f + (T_N/T_F)x} - R\right],\end{aligned} \tag{2.12}$$

Here, r is the nominal dividend payment and defines $R = r/p_f$ as the expected return, with τ interpreted as an adjustment coefficient. Finally, it is noted that when the accumulated market return a_0 becomes less than 0, it indicates the occurrence of a financial bubble burst.

2.1.2 The Model with Safe-Haven Asset

In this section we include the price dynamic of the safe-haven in equation (2.12). Since models of safe-haven assets are still relatively rare nowadays, we adopt the assumption that the change in return of a safe-haven asset, denoted by s, follows a mean-reverting process, as discussed in [9]. To develop such a model according to our initial definition, it is essential to understand the term "safe-haven." The term "safe-haven" refers to a place chosen by living beings to avoid or reduce potential damage to something valuable to them from impending dangers. For investors, "safe-haven" means an alternative investment strategy chosen to protect their assets or their value from economic uncertainties or potential crises. Simply stating that investor behavior changes from buying to selling or vice versa may be too simplistic and rigid. However, evidence from two studies, one by Macro Tronzano [11] and another by Dirk G. Baur [4], indicates that during periods of economic uncertainty, investors tend to shift their cherished risky assets or underlying assets to safe-haven assets. Despite this, there is still a lack of data on the extent of this behavior. To align with [2], we hypothesize that the extent of investor behavior also depends on current economic factors. Therefore, we introduce $-Ex$ as a factor in our safe-haven asset model, where $E > 0$ represents a basic economic factor influencing investors (akin to a weight factor).

Hence, the safe-haven asset model is given by:

$$\frac{ds}{dt} = \alpha \left[T_S(s_f - s) - Ex\right] \tag{2.13}$$

In this context, α signifies the speed of change of the safe-haven asset, T_S represents the trading volume for the safe-haven asset, and s_f denotes the fundamental price of the safe-haven asset. In this equation, the return of safe-haven tends to decrease as the current financial market booms. Furthermore, considering the economic indicators' involvement with financial bubbles, it becomes imperative to contemplate a new model for the underlying asset. Therefore, we derive the following equation:

$$\frac{dp}{dt} = \beta[xT_N + T_F(p_f - p) + E] \tag{2.14}$$

It is clear that equation (2.13)-(2.14) describe that both are driven by the market sentiment. As a result, the dynamic of accumulative return is adjusted as

$$\frac{da_0}{dt} = \tau \left[\frac{r + \tau^{-1}(T_N/T_F)\dot{x}}{p_f + (T_N/T_F)x + E/T_F} - R\right] \tag{2.15}$$

Here, $R = r/(p_f + E/T_F)$. To complete the model modification, we extend the transition probability by assuming that additional information is also drawn from the safe-haven return with directly but negatively proportional to the change in return of the safe-haven asset. By this assumption, dynamic of safe-haven becomes negatively associated with the price of the focusing asset.

To align with Lux's work and his interpretation, it is reasonable to include $a_3 ds/dt$ as a factor influencing the readiness of profit-seekers to follow the crowd in the market. Recently herd behaviour model, which includes the return of safe-haven assets, is represented by

$$\frac{dx}{dt} = 2v[\tanh(a_0 + a_2x + a_3\dot{s}) - x]\cosh(a_0 + a_2x + a_3\dot{s}), \tag{2.16}$$

where a_3 is a weight coefficient that expresses the strength of the safe-haven asset's influence on herd behavior, and $\dot{s}$ represents the rate of change in s. However, this assumption can be considered as two possibilities that is the relationship between two assets can be discrete and continuous. Thus, it is reasonable to separate the model into two sub-models as follows.

Model 1: Continuous Relationship

The first system of equations for the financial bubble model we are considering is represented by the following equation:

$$\begin{aligned} \frac{dx}{dt} &= 2v[\tanh(a_0 + a_2 x + a_3\dot{s}) - x]\cosh(a_0 + a_2 x + a_3\dot{s}), \\ \frac{da_0}{dt} &= \tau\left[\frac{r + \tau^{-1}(T_N/T_F)\dot{x}}{p_f + (T_N/T_F)x + E/T_F} - R\right], \\ \frac{ds}{dt} &= \alpha\left[T_S(s_f - s) - Ex\right], \end{aligned} \tag{2.17}$$

Model (2.17) indicates that investors' behavior is not solely influenced by the actions of the majority but is also affected by accumulated returns and the return information of safe-haven assets when making investment decisions in speculative markets. Investors are aware of this information at all times. This is worth mentioning as it connects to the subsequent model we will consider next.

Model 2: Discrete Relationship

In order to align more closely with the safe-haven asset's definition we have discussed. We now define function $A(a_0)$,

$$A(a_0) = \begin{cases} 0, & \text{if } a_0 \geq 0 \\ 1, & \text{if } a_0 < 0 \end{cases}$$

Incorporating the term $A(a_0)$ to refine and adjust equation (2.17) would enhance the system to adhere more closely to the defined definition. The influence of the returns of safe-haven assets on market participants' decision-making would come into play only when the value of a_0, or the accumulated actual return of the market is negative. Now we have

$$\frac{dx}{dt} = 2v[\tanh(a_0 + a_2x + a_3\dot{s}A(a_0)) - x]\cosh(a_0 + a_2x + a_3\dot{s}A(a_0)). \tag{2.18}$$

where a_3 is defined as the same as the previous model. Therefore, the present models are (2.18)

2.1.3 Stability Analysis

In this section, we aim to determine the (local) stability condition for the equilibrium point of model (2.17). This is because when considering the definition of a bubble as a transient situation where prices oscillate around the fundamental price, analyzing the stability of the system becomes an important aspect. As the bubble may occur when the system undergoes the unstable equilibrium state, the derived condition can be used to confine the parameter space for further investigation.

To determine the equilibrium point of the system (2.17), we first put $dx/dt = 0$, $da_0/dt = 0$, and $ds/dt = 0$, respectively. This is true after the truth that $dx/dt = 0$ and $x = 0$. So, we consider only remained two equations. We also observe that $x = 0$ is only solution for the first equation. Hence, $s = s_f$ is a result. Therefore, we can conclude that our system inherently possesses a unique equilibrium $E(x, a_0, s) = E(0, 0, s_f)$, representing a scenario where the majority of dispositions are balanced, actual returns are zero, and the price of the safe-haven asset equals its fundamental price. For assessing system stability, we rely on the Routh-Hurwitz stability criterion, a mathematical test that is a necessary and sufficient condition for the stability of a linear time-invariant dynamical system, [6] with the following Jacobian matrix:

$$J = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ -\alpha E & 0 & \alpha T_S \end{bmatrix}, \tag{2.19}$$

where $F_{11} = 2vM_2(1 - K_1)M_4 + 2vK_1M_1M_3$,

$F_{12} = 2vT_5 + 2vM_1M_3,$

$F_{13} = -K_2M_5 - K_2M_1M_3,$

$F_{21} = -\frac{K_3T_N(2vT_FM_2 - 2vT_FK_1(1+M_2)M_4 + K_3\tau(r+2M_2M_3T_Nv))}{K_4T_F},$

$F_{22} = \tau K_3(\frac{2vT_NM_5 + 2vT_NM_1M_3}{K_4}),$

$F_{23} = \tau K_3(\frac{T_NK_2M_3 - T_NK_2M_1M_3}{K_4}),$

$K_1 = a_2 - a_3\alpha E,$

$K_2 = 2a_3\alpha T_S v,$

$K_3 = 1/(p_f + E/T_F + xT_N/T_F),$

$K_4 = \tau T_F,$

$M_1 = \sinh(a_0 + a_2x + a_3\alpha((-s + s_f)T_S - Ex)),$

$M_2 = \cosh(a_0 + a_2x + a_3\alpha((-s + s_f)T_S - Ex)),$

$M_3 = -x + \tanh(a_0 + a_2x + a_3\alpha((-s + s_f)T_S - Ex)),$

$M_4 = \text{sech}(a_0 + a_2x + a_3\alpha((-s + s_f)T_S - Ex))^2$

and $M_5 = \text{sech}(a_0 + a_2x + a_3\alpha((-s + s_f)T_S - Ex)).$

After computing the Jacobian matrix, we proceed to analyze the stability of the system at equilibrium points examining the eigenvalues of the characteristic equation. The characteristic equation is obtained from the determinant of the Jacobian matrix subtracted by a scalar multiple of the identity matrix, given by:

$$\det(J - \lambda I) = 0 \tag{2.20}$$

This characteristic polynomial is typically expressed as:

$$a_n\lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0 = 0 \tag{2.21}$$

To determine the stability of the system, we use the Routh-Hurwitz criterion, which involves constructing the Routh-Hurwitz array from the coefficients of the characteristic polynomial. The Routh-Hurwitz array is constructed as follows:

$$
\begin{array}{c|cccc}
s^n & a_n & a_{n-2} & a_{n-4} & \cdots \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\
s^{n-2} & b_1 & b_2 & b_3 & \cdots \\
s^{n-3} & c_1 & c_2 & c_3 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
s^1 & d_1 & d_2 & d_3 & \cdots \\
s^0 & a_0 & 0 & 0 & \cdots
\end{array}
$$

where the elements $b_i, c_i, \ldots$ are computed as follows:

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}, \quad b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}, \quad \text{and so on.}$$

$$c_1 = \frac{b_1 a_{n-1} - a_{n-1} b_2}{b_1}, \quad c_2 = \frac{b_1 a_{n-3} - a_{n-1} b_3}{b_1}, \quad \text{and so on.}$$

Once the Routh-Hurwitz array is constructed, the stability of the system can be determined by examining the first column of the array. If all elements in the first column are positive and there are no sign changes, the system is stable. If there are sign changes, the number of sign changes corresponds to the number of eigenvalues with positive real parts, indicating instability.

After computing the coefficients of the characteristic equation of our differential equation system and constructing the Routh-Hurwitz array, we identified the stability conditions as follows: the equilibrium is stable if and only if either $a < 0$ and $b < 0$. The values of a and b are determined as follows:

$$
\begin{aligned}
a &= -\alpha T_S + 2vC - 2a_3\alpha E v + \frac{2T_N R v}{r T_F}, \\
b &= 2v\left(\frac{RT_N(\alpha T_S - \tau R)}{rT_F} + \alpha T_S\Big(C + \frac{\tau R^2 T_N}{\alpha r T_F(T_S + 2a_3 E v) - 2v(CrT_F + RT_N)}\Big)\right),
\end{aligned}
\tag{2.22}
$$

where $R = r/(p_f + E/T_F)$ and $C = a_2 - 1$.

Before proceeding to the next section, it's important to acknowledge the scope of our stability analysis. While we have successfully identified conditions under which our system exhibits instability, it's essential to note that our focus has been primarily on understanding fluctuations around the fundamental price. However, it's worth mentioning that determining conditions for periodic events remains an ongoing challenge. Despite this limitation, our analysis provides valuable insights into the behavior of our system within the context of instability.

Chapter 3

Results

3.1 Results

In the context of financial bubble phenomena, two factors can indicate its severity. First is its size, which refers to the magnitude of its price fluctuations around the fundamental value, represented by the height from crest to trough. The second factor is its duration, which represents the time it takes for the price fluctuations to complete one cycle, indicated by the length from crest to crest.

We have omitted the analysis of events in the early stages of the mechanism concerning size and duration in both (2.17) and (2.18) due to their non-periodic nature. Instead, we focus on the analysis of events in the second stage when the system exhibits periodic solutions, as depicted in Figure 3.1.

In this section, we calculate the two indicators from the numerical solutions of the models using the parameter values in Table 3.1. To verify whether the results align with our hypothesis, which posits that the inclusion of information from safe-haven assets reduces the severity of financial bubbles which are temporary events where asset prices deviate and follow with the fluctuation around their fundamental value, we consider the stability conditions outlined in the previous section. Given the unique equilibrium point of the system, it is sufficient to select parameters that induce instability in system (2.17) for this analysis.

Parameter	Description	Value(unit)
a_2	Strength of herd behavior	1.125
a_3	Strength of safe-haven asset	1.25
r	Constant nominal dividend payment	1.0
T_N	Trading volume of speculative investor	21/160
T_F	Trading volume of fundamental investor	3/4
p_F	Fundamental price of underlying asset	7/10
T_S	Trading volume of safe-haven asset	1.0
s_F	Fundamental price of safe-haven asset	1.3
α	Speed of change on safe-haven asset	1.0
β	Speed of change on underlying asset	1.0
E	Economic factor	0.02
v	Speed of change on probability	0.5
τ	Adjustment coefficient	1.0

Table 3.1: Parameter values used in numerical calculations.

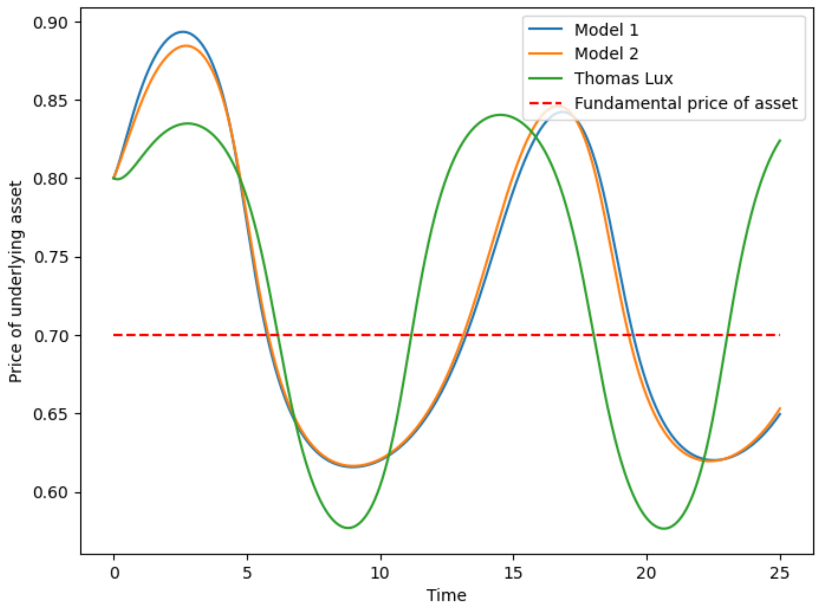


Figure 3.1: Sample of price dynamics/movements of the underlying asset for each model with a set of initial conditions $p = 0.8$, $x = 0.5$, $a_0 = 1$ and $s = 1$.

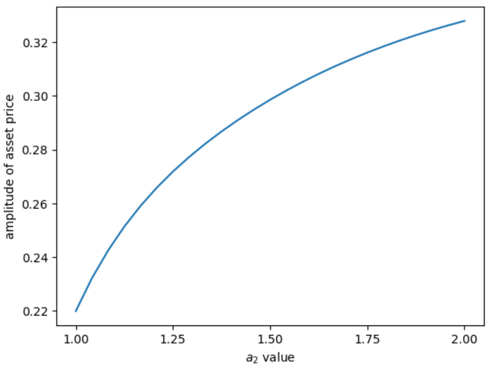
3.1.1 Model 1's Result

According to model (2.17), it demonstrates how safe-haven assets play a role in investors' decision-making at all times. When considering the weight factor variable a_3, which represents the weight that profit-seekers give to information about safe-haven assets, from

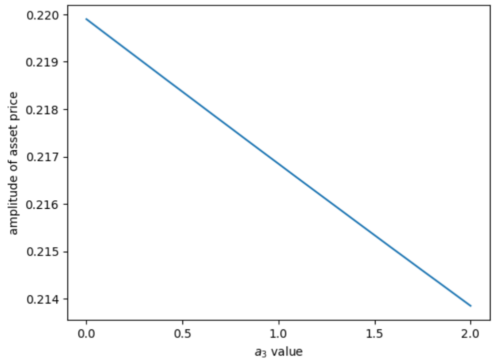
Figure 3.2b, it can be observed that as a_3 increases, the height of the bubble decreases. In this scenario, we might argue that when profit-seekers who exhibit herding behavior take a moment to observe information from safe-haven assets before considering buying/selling the underlying asset they are interested in, in cases where these profit-seekers make mistakes in their decision-making, it may help reduce the resulting losses.

As for the weight factor variable a_2, which represents the weight that profit-seekers give to the noise of the crowd before considering buying/selling the underlying asset they are interested in, from Figure 3.2a, it can be observed that as a_2 increases, the height of the bubble also increases. It is evident that when people are ready to make decisions to buy/sell the underlying asset solely because others are doing so, it is not surprising that the price of this asset may soar to the sky or plummet underground.

Upon examining financial bubble in term of the duration in Figures 3.3a and 3.3b, both variables a_2 and a_3 yield similar results. That is, as these variables increase, the duration of price fluctuations around the fundamental value for one cycle also increases. This may be beneficial as it suggests a decrease in market volatility.

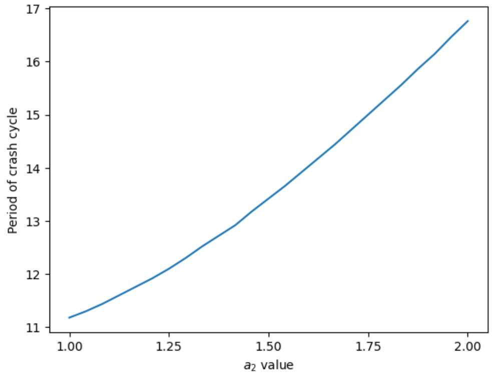


(a) Impact of a_2 on asset's amplitude as a_3 is 1.

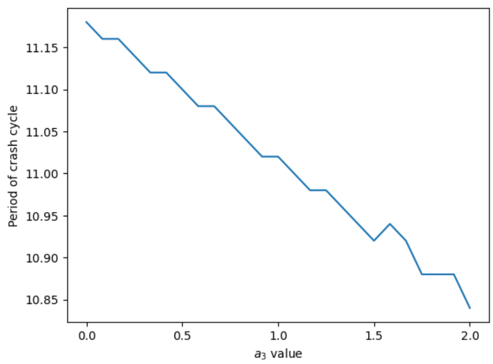


(b) Impact of a_3 on asset's amplitude as a_2 is 1.

Figure 3.2: Impacts of a_2 and a_3 on underlying asset's amplitude of model 1 as a_3 is 1 and a_2 is 1 respectively.

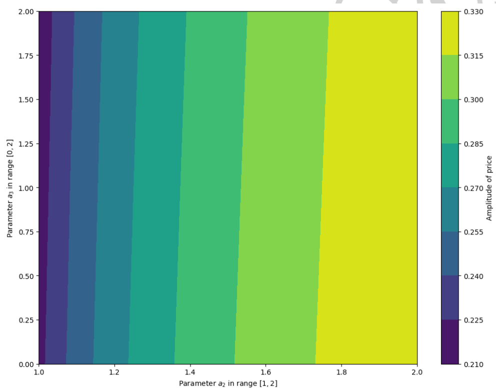


(a) Impact of a_2 on asset's period as a_3 is 1.

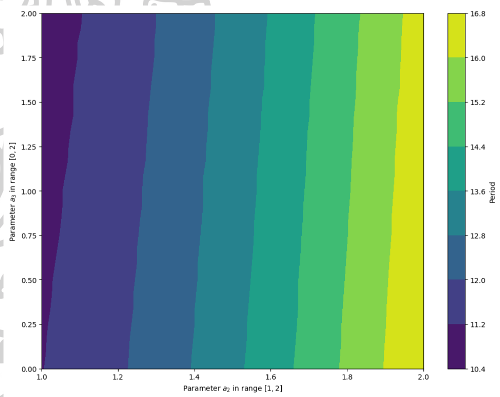


(b) Impact of a_3 on asset's period as a_2 is 1.

Figure 3.3: Impacts of a_2 and a_3 on underlying asset's period of model 1 as a_3 is 1 and a_2 is 1 respectively.



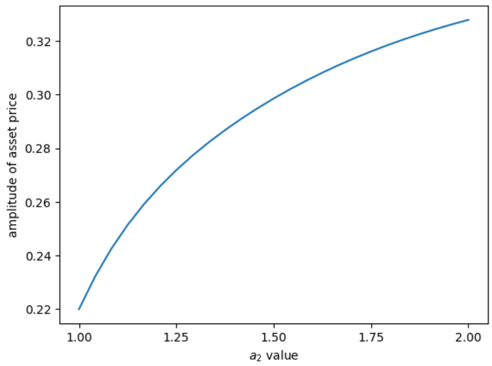
(a) Impacts of a_2 and a_3 on underlying asset's amplitude.



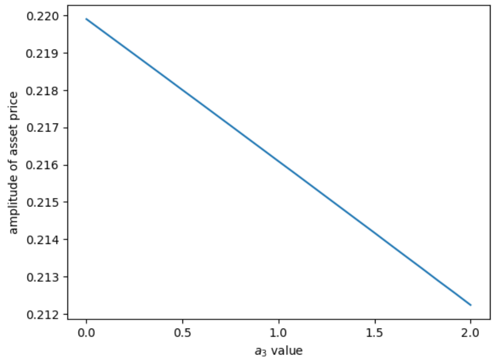
(b) Impacts of a_2 and a_3 on underlying asset's period.

Figure 3.4: Impacts of a_2 and a_3 on underlying asset's price of model 1 in contour plot.

Figures 3.4a and 3.4b represent contour plots illustrating the impacts of a_2 and a_3 on the underlying asset's price in terms of amplitude and period in Model 1. We have seen that the change in combination of two parameters does not make significant change of the amplitudes and periods from the pattern when fixing one parameter. The results are more relatively sensitive to the change of a2 than a3. The contour plot shows that the safe-haven strategy and the herding behavior are uncorrelated.

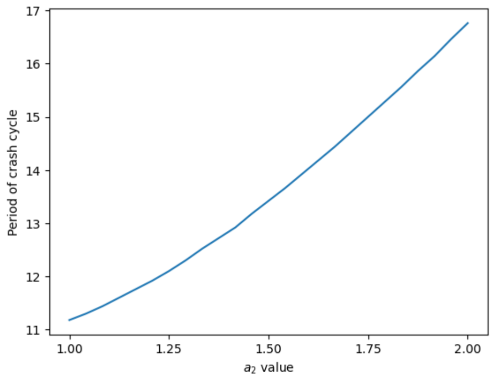


(a) Impact of a_2 on asset's amplitude as a_3 is 1.

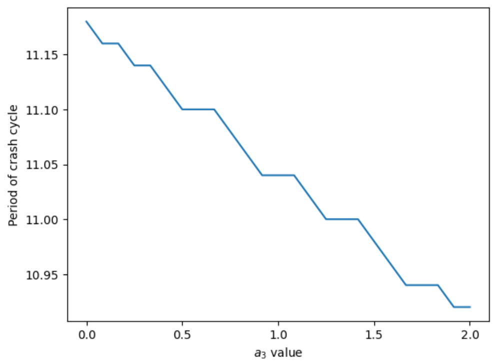


(b) Impact of a_3 on asset's amplitude as a_2 is 1.

Figure 3.5: Impacts of a_2 and a_3 on underlying asset's amplitude of model 2 as a_3 is 1 and a_2 is 1 respectively.



(a) Impact of a_2 on asset's period as a_3 is 1.

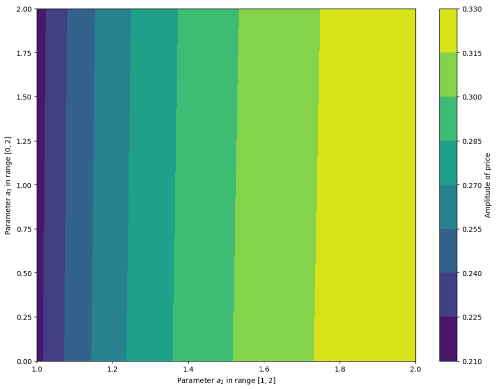


(b) Impact of a_3 on asset's period as a_2 is 1.

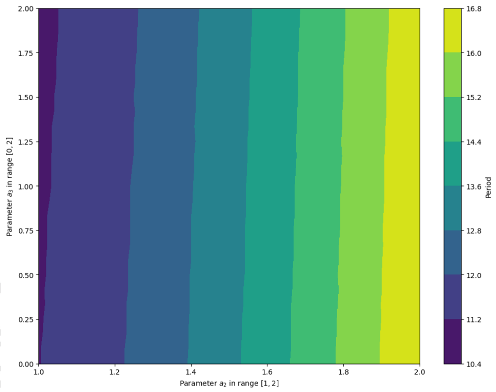
Figure 3.6: Impacts of a_2 and a_3 on underlying asset's period of model 2 as a_3 is 1 and a_2 is 1 respectively.

3.1.2 Model 2's Result

For the results of model (2.18), where we stated that safe-haven assets play a role in investors' decision-making only when the market enters a crisis or downturn, as indicated by the actual return a_0 being less than 0, the outcomes, whether in terms of amplitude as shown in figures 3.5a and 3.5b, or in terms of period as shown in figures 3.6a and 3.6b, yield similar Model 1(a continuous relationship) both numerical result and interpretations. However, when comparing the outcomes of both models from both the amplitude and period perspectives by the effect from a_3, it is evident that Model 1 provides better results in both aspects, as depicted in figures 3.8a and 3.8b. Therefore, we can conclude that investors' continuous interest in safe-haven assets at all times leads to less market volatility compared to when they only pay attention to them during crisis periods.



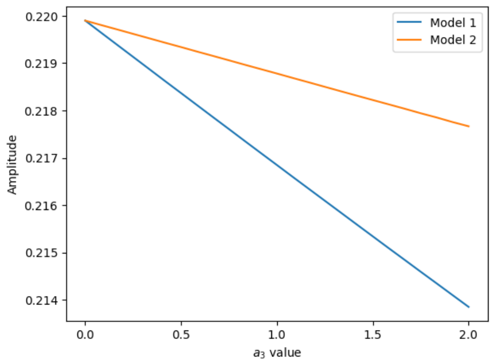
(a) Impacts of a_2 and a_3 on underlying asset's amplitude.



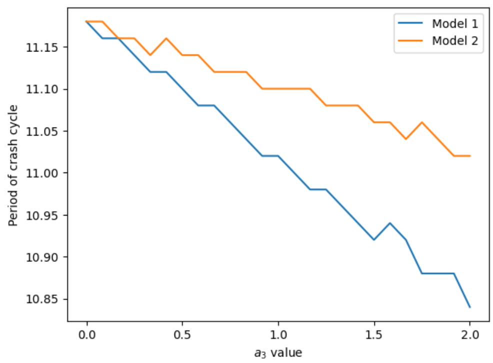
(b) Impacts of a_2 and a_3 on underlying asset's period.

Figure 3.7: Impacts of a_2 and a_3 on underlying asset's price of model 2 in contour plot.

Figures 3.7a and 3.7b represent contour plots illustrating the impacts of a_2 and a_3 on the underlying asset's price in terms of amplitude and height in Model 2.



(a) Comparison between model 1&2 on amplitude from impact of a_3



(b) Comparison between model 1&2 on period from impact of a_3

Figure 3.8: Comparison between model 1&2 on amplitude and period from impact of a_3

Chapter 4

Conclusion

4.1 Conclusion

Our findings from the safe-haven asset model, as illustrated by the Mean Reversion model, confirm our underlying assumption in both respects. Compare to Lux's, this indicates that when investors base their decisions to buy or sell the underlying asset on information beyond mere consensus, it can reduce market volatility. In this context, volatility refers to the intensity of financial bubbles. Or in other words, wisdom prevails over emotions.

We have proposed the mechanistic models extended from the previous work. The model composes of the dynamics of disposition variables, the accumulative difference of returns and safe-haven asset. The present model always has only one equilibrium point. The stability conditions are more complicate than of the previous work since the number of parameters of safe-haven asset are added. However, the common necessary conditions are that $a_2 < 1$. This implies that the onset of financial bubble requires the strong influence of herding behavior.

Understanding the existence of financial bubbles and being able to explain them in another form, as we have proposed, would be beneficial for analyzing whether the current situation warrants diversification of our investment risks or not. In addition to that, safe-haven assets are likely to be another option for hedging or portfolio allocation. Since the parameters used in our experiments are not specified, it may be possible to consider the proportions of holding safe-haven assets for hedging or portfolio allocation.

This research, while explaining the influence of safe-haven assets on financial bubbles in a deterministic form, also paves the way for exploring stochastic models. This extension could encompass various aspects, including price prediction models or financial bubble models, sentiment analysis of profit seekers in the market, or expressing it in other forms. There are numerous avenues to explore. Another potential direction is to include other assets beyond safe-haven assets to observe the behavior of profit seekers, price movements, and sentiment, which could be beneficial for hedging or portfolio allocation. Undoubtedly, there is much more to investigate.

As mentioned earlier, this paper is an extension of Thomas Lux's work on "Herd behavior, bubbles and crashes". In this regard, it raises the question of what would happen if other assets were involved with the underlying asset, and we chose it as the safe-haven asset. While our proposed safe-haven asset model may not fully capture the characteristics indicative of a safe-haven asset and could prompt questions about its efficacy, this could serve as a starting point for further development of Thomas Lux's model from another interesting perspective.

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